Linear Programming Problems – Formulation

Linear Programming is a mathematical technique for optimum allocation of limited or scarce resources, such as labour, material, machine, money, energy and so on , to several competing activities such as products, services, jobs and so on, on the basis of a given criteria of optimality.

The term **'Linear'** is used to describe the proportionate relationship of two or more variables in a model. The given change in one variable will always cause a resulting proportional change in another variable.

The word , **'programming'** is used to specify a sort of planning that involves the economic allocation of limited resources by adopting a particular course of action or strategy among various alternatives strategies to achieve the desired objective.

Hence, **Linear Programming** is a mathematical technique for optimum allocation of limited or scarce resources, such as labour, material, machine, money energy etc.

Structure of Linear Programming model.

The general structure of the Linear Programming model essentially consists of three components.

i) The activities (variables) and their relationships
ii) The objective function and
iii) The constraints

The activities are represented by X1, X2, X3 ……..Xn.
These are known as Decision variables.

The objective function of an LPP (Linear Programming Problem) is a mathematical representation of the objective in terms a measurable quantity such as profit, cost, revenue, etc.

Optimize (Maximize or Minimize) Z=C1X1 +C2X2+ ………..Cn Xn

Where Z is the measure of performance variable

X1, X2, X3, X4…..Xn are the decision variables
And C1, C2, …Cn are the parameters that give contribution to decision variables.

The constraints These are the set of linear inequalities and/or equalities which impose restriction of the limited resources

Assumptions of Linear Programming

Certainty.

In all LP models it is assumed that, all the model parameters such as availability of resources, profit (or cost) contribution of a unit of decision variable and consumption of resources by a unit of decision variable must be known and constant.

Divisibility (Continuity)

The solution values of decision variables and resources are assumed to have either whole numbers (integers) or mixed numbers (integer or fractional). However, if only integer variables are desired, then Integer programming method may be employed.

Additivity

The value of the objective function for the given value of decision variables and the total sum of resources used, must be equal to the sum of the contributions (Profit or Cost) earned from each decision variable and sum of the resources used by each decision variable respectively. /The objective function is the direct sum of the individual contributions of the different variables

Linearity

All relationships in the LP model (i.e. in both objective function and constraints) must be linear.

General Mathematical Model of an LPP

Optimize (Maximize or Minimize) Z=C1 X1 + C2 X2 +……+CnXn

Subject to constraints,

a11X1+ a 12X2+………………+ a 1nXn ($\leq$,=,$\geq$) b1

a21X1+ a 22X2+………………+ a 2nXn ($\leq$,=,$\geq$) b2

a31X1+ a 32X2+………………+ a 3nXn ($\leq$,=,$\geq$) b3

am1X1+ a m2X2+………………+ a mnXn ($\leq$,=,$\geq$) bm

and X1, X2 ….Xn $\geq$

Guidelines for formulating Linear Programming model

i) Identify and define the decision variable of the problem

ii) Define the objective function

iii) State the constraints to which the objective function should be optimized

(i.e. Maximization or Minimization)

iv) Add the non-negative constraints from the consideration that the negative values of the decision variables do not have any valid physical interpretation

Example 1.

A manufacturer produces two types of models M1 and M2.Each model of the type M1 requires 4 hours of grinding and 2 hours of polishing; where as each model of M2 requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works 60 hours a week. Profit on M1 model is Rs.3.00 and on model M2 is Rs.4.00.Whatever produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models, so that he makes maximum profit in a week?

i) Identify and define the decision variable of the problem
   Let X1 and X2 be the number of units of M1 and M2 model.

ii) Define the objective function

Since the profits on both the models are given, the objective function is to maximize the profit.

Max Z = 3X1 + 4X2

iii) State the constraints to which the objective function should be optimized (i.e. Maximization or Minimization)

There are two constraints one for grinding and the other for polishing.

The grinding constraint is given by

$4X1 + 2X2 \leq 80$

No of hours available on grinding machine per week is 40 hrs. There are two grinders. Hence the total grinding hour available is 40 X 2 = 80 hours.

The polishing constraint is given by

$2X1 + 5X2 \leq 180$

No of hours available on polishing machine per week is 60 hrs. There are three grinders. Hence the total grinding hour available is 60 X 3 = 180 hours.

Finally we have,

Max Z = 3X1 + 4X2

Subject to constraints,

$4X1 + 2X2 \leq 80$

$2X1 + 5X2 \leq 180$

$X1, X2 \geq 0$

Example 2.

A firm is engaged in producing two products. A and B. Each unit of product A requires 2 kg of raw material and 4 labour hours for processing, where as each unit of B requires 3 kg of raw materials and 3 labour hours for the same type. Every week, the firm has an availability of 60 kg of raw material and 96 labour hours. One unit of product A sold yields Rs.40 and one unit of product B sold gives Rs.35 as profit.

Formulate this as an Linear Programming Problem to determine as to how many units of each of the products should be produced per week so that the firm can earn maximum profit.

i) Identify and define the decision variable of the problem

Let X1 and X2 be the number of units of product A and product B produced per week.

ii) Define the objective function

Since the profits of both the products are given,

the objective function is to maximize the profit.

MaxZ = 40X1 + 35X2

iii) State the constraints to which the objective function should be optimized (i.e. Maximization or Minimization)

There are two constraints one is raw material constraint and the other one is labour constraint..

The raw material constraint is given by

$2X1 + 3X2 \leq 60$

The labour hours constraint is given by

$4X1 + 3X2 \leq 96$

Finally we have,

MaxZ = 40X1 + 35X2

Subject to constraints,

$2X1 + 3X2 \leq 60$

$4X1 + 3X2 \leq 96$

$X1, X2 \geq 0$

Example 3.

The agricultural research institute suggested the farmer to spread out atleast 4800 kg of special phosphate fertilizer and not less than 7200 kg of a special nitrogen fertilizer to raise the productivity of crops in his fields. There are two sources for obtaining these – mixtures A and mixtures B. Both of these are available in bags weighing 100kg each and they cost Rs.40 and Rs.24 respectively. Mixture A contains phosphate and nitrogen equivalent of 20kg and 80 kg respectively, while mixture B contains these ingredients equivalent of 50 kg each. Write this as an LPP and determine how many bags of each type the farmer should buy in order to obtain the required fertilizer at minimum cost.

i) Identify and define the decision variable of the problem

Let X1 and X2 be the number of bags of mixture A and mixture B.

ii) Define the objective function

The cost of mixture A and mixture B are given ;

the objective function is to minimize the cost

Min.Z = 40X1 + 24X2

iii) State the constraints to which the objective function should be optimized.

The above objective function is subjected to following constraints.

$20X1 + 50X2 \geq 4800$ Phosphate requirement
$80X1 + 50X2 \geq 7200$ Nitrogen requirement
$X1, X2 \geq 0$

Finally we have,
Min.Z = 40X1 + 24X2
is subjected to three constraints
$20X1 + 50X2 \geq 4800$
$80X1 + 50X2 \geq 7200$
$X1, X2 \geq 0$

Example 4.
A firm can produce 3 types of cloth, A , B and C.3 kinds of wool are required Red, Green and Blue.1 unit of length of type A cloth needs 2 meters of red wool and 3 meters of blue wool.1 unit of length of type B cloth needs 3 meters of red wool, 2 meters of green wool and 2 meters of blue wool.1 unit type of C cloth needs 5 meters of green wool and 4 meters of blue wool. The firm has a stock of 8 meters of red, 10 meters of green and 15 meters of blue. It is assumed that the income obtained from 1 unit of type A is Rs.3, from B is Rs.5 and from C is Rs.4.Formulate this as an LPP.**(December2005/January 2006)**

i) Identify and define the decision variable of the problem
Let X1, X2 and X3 are the quantity produced of cloth type A,B and C respectively.
ii) Define the objective function
The incomes obtained for all the three types of cloths are given; the objective function is to maximize the income.
Max Z = 3X1 + 5X2 + 4X3
iii) State the constraints to which the objective function should be optimized.
The above objective function is subjected to following three constraints.
$2X1 + 3X2 \leq 8$
$2X2 + 5X3 \leq 10$
$3X1 + 2X2 + 4X3 \leq 15$
$X1, X2\ X3 \geq 0$

Finally we have,
Max Z = 3X1 + 5X2 + 4X3
is subjected to three constraints
$2X1 + 3X2 \leq 8$
$2X2 + 5X3 \leq 10$
$3X1 + 2X2 + 4X3 \leq 15$
$X1, X2\ X3 \geq 0$

Example 5.
A Retired person wants to invest upto an amount of Rs.30,000 in fixed income securities. His broker recommends investing in two Bonds: Bond A yielding 7% and Bond B yielding 10%. After some consideration, he decides to invest at most of Rs.12,000 in bond B and atleast Rs.6,000 in Bond A. He also wants the amount invested in Bond A to be atleast equal to the amount invested in Bond B. What should the broker recommend if the investor wants to maximize his return on investment? Solve graphically.
(January/February 2004)

i) Identify and define the decision variable of the problem
Let X1 and X2 be the amount invested in Bonds A and B.
ii) Define the objective function
Yielding for investment from two Bonds are given; the objective function is to maximize the yielding.
Max Z = 0.07X1 + 0.1X2
iii) State the constraints to which the objective function should be optimized.
The above objective function is subjected to following three constraints.
$X1 + X2 \leq 30{,}000$
$X1 \geq 6{,}000$
$X2 \leq 12{,}000$
$X1 - X2 \geq 0$
$X1, X2 \geq 0$

Finally we have,
MaxZ = 0.07X1 + 0.1X2
is subjected to three constraints
$X1 + X2 \leq 30{,}000$
$X1 \geq 6{,}000$
$X2 \leq 12{,}000$
$X1 - X2 \geq 0$
$X1, X2 \geq 0$

Minimization problems
Example 5.
A person requires 10, 12, and 12 units chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A,B and C respectively per jar. A dry product contains 1,2 and 4 units of A,B and C per carton.
If the liquid product sells for Rs.3 per jar and the dry product sells for Rs.2 per carton, how many of each should be purchased, in order to minimize the cost and meet the requirements?

i) Identify and define the decision variable of the problem
Let X1 and X2 be the number of units of liquid and dry products.
ii) Define the objective function
The cost of Liquid and Dry products are given ; the objective function is to minimize the cost

Min. $Z = 3X1 + 2X2$

iii) State the constraints to which the objective function should be optimized.
The above objective function is subjected to following three constraints.

$5X1 + X2 \geq 10$
$2X1 + 2X2 \geq 12$
$X1 + 4X2 \geq 12$
$X1, X2 \geq 0$

Finally we have,
Min. $Z = 3X1 + 2X2$
is subjected to three constraints
$5X1 + X2 \geq 10$
$2X1 + 2X2 \geq 12$
$X1 + 4X2 \geq 12$
$X1, X2 \geq 0$

Example 6.

A Scrap metal dealer has received a bulk order from a customer for a supply of atleast 2000 kg of scrap metal. The consumer has specified that atleast 1000 kgs of the order must be high quality copper that can be melted easily and can be used to produce tubes. Further, the customer has specified that the order should not contain more than 200 kgs of scrap which are unfit for commercial purposes. The scrap metal dealer purchases the scrap from two different sources in an unlimited quantity with the following percentages (by weight) of high quality of copper and unfit scrap

	Source A	Source B
Copper	40%	75%
Unfit Scrap	7.5%	10%

The cost of metal purchased from source A and source B are Rs.12.50 and Rs.14.50 per kg respectively. Determine the optimum quantities of metal to be purchased from the two sources by the metal scrap dealer so as to minimize the total cost **(February 2002)**

i) Identify and define the decision variable of the problem
Let X1 and X2 be the quantities of metal to be purchased from the two sources A and B.
ii) Define the objective function
The cost of metal to be purchased by the metal scrap dealer are given;
the objective function is to minimize the cost
Min. $Z = 12.5X1 + 14.5X2$
iii) State the constraints to which the objective function should be optimized.
The above objective function is subjected to following three constraints.

$X1 + X2 \geq 2{,}000$
$0.4X1 + 0.75X2 \geq 1{,}000$
$0.075X1 + 0.1X2 + 4X3 \leq 200$
$X1, X2 \geq 0$

Finally we have,
Min. $Z = 12.5X1 + 14.5X2$
is subjected to three constraints
$X1 + X2 \geq 2,000$
$0.4X1 + 0.75X2 \geq 1,000$
$0.075X1 + 0.1X2 + 4X3 \leq 200$
$X1, X2 \geq 0$

Example 7.
A farmer has a 100 acre farm. He can sell all tomatoes, lettuce or radishes and can raise the price to obtain Rs.1.00 per kg. for tomatoes , Rs.0.75 a head for lettuce and Rs.2.00 per kg for radishes. The average yield per acre is 2000kg.of tomatoes, 3000 heads of lettuce and 1000 kgs of radishes. Fertilizers are available at Rs.0.50 per kg and the amount required per acre is 100 kgs for each tomatoes and lettuce and 50kgs for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour are available at Rs.20.00 per man-day. Formulate this problem as LP model to maximize the farmers profit.

i) Identify and define the decision variable of the problem
Let X1 and X2 and X3 be number acres the farmer grows tomatoes, lettuce and radishes respectively.
ii) Define the objective function
The objective of the given problem is to maximize the profit.
The profit can be calculated by subtracting total expenditure from the total sales
Profit = Total sales – Total expenditure

The farmer produces 2000X1 kgs of tomatoes, 3000X2 heads of lettuce, 1000X3 kgs of radishes.
Therefore the total sales of the farmer will be
$= \text{Rs.} (1 \times 2000X1 + 0.75 \times 3000X2 + 2 \times 100X3)$
Total expenditure (fertilizer expenditure) will be
$= \text{Rs.}20 (5X1 + 6X2 + 5X3)$
Farmer's profit will be
$Z = (1 \times 2000X1 + 0.75 \times 3000X2 + 2 \times 100X3) –$
$\{ [0.5 \times 100 \times X1+0.5 \times 100 \times X2 + 50X3]+ [20 \times 5 \times X1+20 \times 6 \times X2 + 20 \times 5 \times X3]\}$
$=1850X1 + 2080X2 + 1875X3$

Therefore the objective function is
Maximise $Z = 1850X1 + 2080X2 + 1875X3$

iii) State the constraints to which the objective function should be optimized.
The above objective function is subjected to following constraints.
Since the total area of the firm is 100 acres
$X1 + X2 + X3 \leq 100$
The total man-days labour is 400 man-days

$5X1 + 6X2 + 5X3 \leq 400$
Finally we have,
Maximise $Z = 1850X1 + 2080X2 + 1875X3$
is subjected to three constraints
$X1 + X2 + X3 \leq 100$
$5X1 + 6X2 + 5X3 \leq 400$
$X1, X2\ X3 \geq 0$

Example 8.
An electronics company produces three types of parts for automatic washing machines .It purchases castings of the parts from a local foundry and then finishes the part on drilling, shaping and polishing machines. The selling prices of parts A, B, and C respectively are Rs 8, Rs.10 and Rs.14.All parts made can be sold. Castings for parts A, B and C respectively cost Rs.5, Rs.6 and Rs.10.
The shop possesses only one of each type of machine. Cost per hour to run each of the three machines are Rs.20 for drilling, Rs.30 for shaping and Rs.30 for polishing. The capacities (parts per hour) for each part on each machine are shown in the following table.

Machine	Capacities Per Hour		
	Part A	Part B	Part C
Drilling	25	40	25
Shaping	25	20	20
Polishing	40	30	40

The management of the shop wants to know how many parts of each type it should produce per hour in order to maximize profit for an hour's run. Formulate this problem as an LP model so as to maximize total profit to the company.

i) Identify and define the decision variable of the problem
Let X1 and X2 and X3 be the number of types A, B and C parts produced per hour respectively .
ii) Define the objective function
With the information given, the hourly profit for part A, B, and C would be as follows
Profit per type A part = $(8 – 5) – (20/25 + 30/25 + 30/40) = 0.25$
Profit per type B part = $(10 – 6) – (20/40 + 30/20 + 30/30) = 1$
Profit per type C part = $(14 – 10) – (20/25 + 30/20 + 30/40) = 0.95$
Then,
Maximize $Z = 0.25\ X1 + 1X2 + 0.95X3$

iii) State the constraints to which the objective function should be optimized.
The above objective function is subjected to following constraints.

i) The drilling machine constraint
$X1/25 + X2/40 + X3/24 \leq 1$
ii) The shaping machine constraint
$X1/25 + X2/20 + X3/20 \leq 1$
iii) The polishing machine constraint
$X1/40 + X2/30 + X3/40 \leq 1$
$X1, X2, X3 \geq 0$

Finally we have,
Maximize $Z = 0.25\ X1 + 1X2 + 0.95X3$
Subject to constraints
$X1/25 + X2/40 + X3/24 \leq 1$
ii) The shaping machine constraint
$X1/25 + X2/20 + X3/20 \leq 1$
iii) The polishing machine constraint
$X1/40 + X2/30 + X3/40 \leq 1$
$X1, X2, X3 \geq 0$

Example 9.
A city hospital has the following minimal daily requirements for nurses.

Period	Clock time (24 hours day)	Minimum number of nurses required
1	6 a.m. – 10 a.m.	2
2	10 a.m. – 2 p.m.	7
3	2 p.m. – 6 p.m.	15
4	6 p.m. – 10 p.m.	8
5	10 p.m. – 2 a.m.	20
6	2 a.m. – 6 a.m.	6

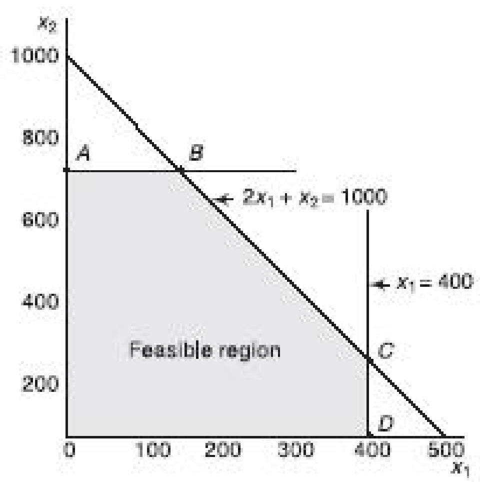
Nurses report at the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the minimal number of nurses to be employed so that there will be a sufficient number of nurses available for each period.
Formulate this as a linear programming problem by setting up appropriate constraints and objective function.

i) Identify and define the decision variable of the problem
Let X1, X2, X3, X4, X5 and X6 be the number of nurses joining duty at the beginning of periods 1, 2, 3, 4, 5 and 6 respectively.
ii) Define the objective function
Minimize $Z = X1 + X2 + X3 + X4 + X5 + X6$
iii) State the constraints to which the objective function should be optimized.
The above objective function is subjected to following constraints.

$X1 + X2 \geq 7$
$X2 + X3 \geq 15$
$X3 + X4 \geq 8$
$X4 + X5 \geq 20$
$X5 + X6 \geq 6$
$X6 + X1 \geq 2$
$X1, X2, X3, X4, X5, X6 \geq 0$

Linear Programming: Graphical Solution
Example 1.
Solve the following LPP by graphical method
Maximize $Z = 5X1 + 3X2$
Subject to constraints
$2X1 + X2 \leq 1000$
$X1 \leq 400$
$X1 \leq 700$
$X1, X2 \geq 0$

Solution:
The first constraint $2X1 + X2 \leq 1000$ can be represented as follows.
We set $2X1 + X2 = 1000$
When $X1 = 0$ in the above constraint, we get,
$2 \times 0 + X2 = 1000$
$X2 = 1000$
Similarly when $X2 = 0$ in the above constraint, we get,
$2X1 + 0 = 1000$
$X1 = 1000/2 = 500$
The second constraint $X1 \leq 400$ can be represented as follows,
We set $X1 = 400$
The third constraint $X2 \leq 700$ can be represented as follows,
We set $X2 = 700$



The constraints are shown plotted in the above figure

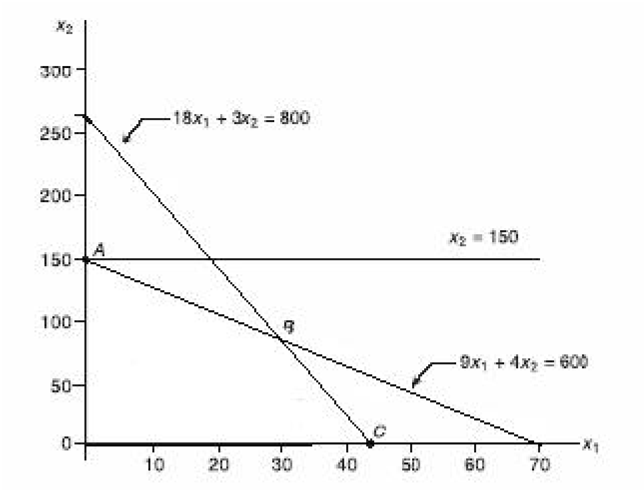
Point	X1	X2	Z = 5X1 +3X2
0	0	0	0
A	0	700	Z = 5 x 0 + 3 x 700 = 2,100
B	150	700	Z = 5 x 150 + 3 x 700 = 2,850* Maximum
C	400	200	Z = 5 x 400 + 3 x 200 = 2,600
D	400	0	Z = 5 x 400 + 3 x 0 = 2,000

The Maximum profit is at point B
When X1 = 150 and X2 = 700
Z = <u>2850</u>

Example 2.
Solve the following LPP by graphical method
Maximize Z = 400X1 + 200X2
Subject to constraints
18X1 + 3X2 ≤ 800
9X1 + 4X2 ≤ 600
X2 ≤ 150
X1, X2 ≥ 0

Solution:
The first constraint 18X1 + 3X2 ≤ 800 can be represented as follows.
We set 18X1 + 3X2 = 800
When X1 = 0 in the above constraint, we get,
18 x 0 + 3X2 = 800
X2 = 800/3 = 266.67
Similarly when X2 = 0 in the above constraint, we get,
18X1 + 3 x 0 = 800
X1 = 800/18 = 44.44

The second constraint 9X1 + 4X2 ≤ 600 can be represented as follows,
We set 9X1 + 4X2 = 600
When X1 = 0 in the above constraint, we get,
9 x 0 + 4X2 = 600
X2 = 600/4 = 150
Similarly when X2 = 0 in the above constraint, we get,
9X1 + 4 x 0 = 600
X1 = 600/9 = 66.67
The third constraint X2 ≤ 150 can be represented as follows,
We set X2 = 150



Point	X1	X2	Z = 400X1 + 200X2
0	0	0	0
A	0	150	Z = 400 x 0+ 200 x 150 = 30,000* Maximum
B	31.11	80	Z = 400 x 31.1 + 200 x 80 = 28,444.4
C	44.44	0	Z = 400 x 44.44 + 200 x 0 = 17,777.8

The Maximum profit is at point A
When X1 = 150 and X2 = 0
Z = <u>30,000</u>

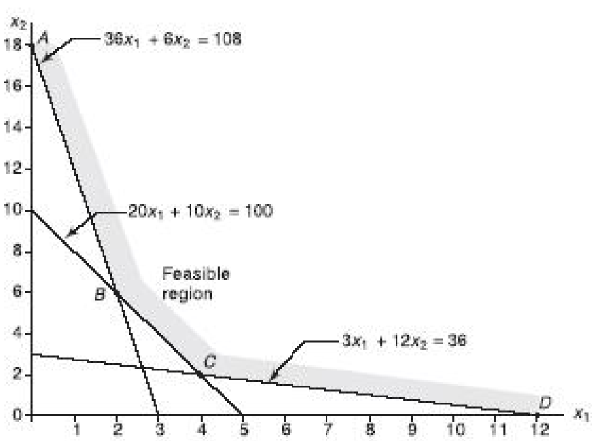
Example 3.
Solve the following LPP by graphical method
Minimize Z = 20X1 + 40X2
Subject to constraints
36X1 + 6X2 ≥ 108
3X1 + 12X2 ≥ 36
20X1 + 10X2 ≥ 100
X1 X2 ≥ 0

Solution:
The first constraint 36X1 + 6X2 ≥ 108 can be represented as follows.
We set 36X1 + 6X2 = 108
When X1 = 0 in the above constraint, we get,

36 x 0 + 6X2 = 108
X2 = 108/6 = 18
Similarly when X2 = 0 in the above constraint, we get,
36X1 + 6 x 0 = 108
X1 = 108/36 = 3

The second constraint3X1 + 12X2 ≥ 36 can be represented as follows,
We set 3X1 + 12X2 = 36
When X1 = 0 in the above constraint, we get,
3 x 0 + 12X2 = 36
X2 = 36/12 = 3
Similarly when X2 = 0 in the above constraint, we get,
3X1 + 12 x 0 = 36
X1 = 36/3 = 12

The third constraint20X1 + 10X2 ≥ 100 can be represented as follows,
We set 20X1 + 10X2 = 100
When X1 = 0 in the above constraint, we get,
20 x 0 + 10X2 = 100
X2 = 100/10 = 10
Similarly when X2 = 0 in the above constraint, we get,
20X1 + 10 x 0 = 100
X1 = 100/20 = 5



Point	X1	X2	Z = 20X1 + 40X2
0	0	0	0
A	0	18	Z = 20 x 0 + 40 x 18 = 720
B	2	6	Z = 20 x2 + 40 x 6 = 280
C	4	2	Z = 20 x 4 + 40 x 2 = 160* Minimum
D	12	0	Z = 20 x 12 + 40 x 0 = 240

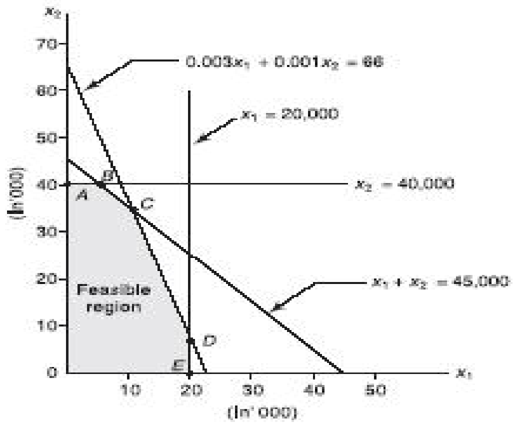
The Minimum cost is at point C
When $X1 = 4$ and $X2 = 2$
$Z = 160$

Example 4.
Solve the following LPP by graphical method
Maximize $Z = 2.80X1 + 2.20X2$
Subject to constraints
$X1 \leq 20{,}000$
$X2 \leq 40{,}000$
$0.003X1 + 0.001X2 \leq 66$
$X1 + X2 \leq 45{,}000$
$X1\ X2 \geq 0$

Solution:
The first constraint $X1 \leq 20{,}000$ can be represented as follows.
We set $X1 = 20{,}000$
The second constraint $X2 \leq 40{,}000$ can be represented as follows,
We set $X2 = 40{,}000$

The third constraint $0.003X1 + 0.001X2 \leq 66$ can be represented as follows,
We set $0.003X1 + 0.001X2 = 66$
When $X1 = 0$ in the above constraint, we get,
$0.003 \times 0 + 0.001X2 = 66$
$X2 = 66/0.001 = 66{,}000$
Similarly when $X2 = 0$ in the above constraint, we get,
$0.003X1 + 0.001 \times 0 = 66$
$X1 = 66/0.003 = 22{,}000$

The fourth constraint $X1 + X2 \leq 45{,}000$ can be represented as follows,
We set $X1 + X2 = 45{,}000$
When $X1 = 0$ in the above constraint, we get,
$0 + X2 = 45{,}000$
$X2 = 45{,}000$
Similarly when $X2 = 0$ in the above constraint, we get,
$X1 + 0 = 45{,}000$
$X1 = 45{,}000$



Point	X1	X2	Z = 2.80X1 + 2.20X2
0	0	0	0
A	0	40,000	Z = 2.80 x 0 + 2.20 x 40,000 = 88,000
B	5,000	40,000	Z = 2.80 x 5,000 + 2.20 x 40,000 = 1,02,000
C	10,500	34,500	Z = 2.80 x 10,500 + 2.20 x 34,500 = 1,05,300* Maximum
D	20,000	6,000	Z = 2.80 x 20,000 + 2.20 x 6,000 = 69,200
E	20,000	0	Z = 2.80 x 20,000 + 2.20 x 0 = 56,000

The Maximum profit is at point C
When X1 = 10,500 and X2 = 34,500
Z = 1,05,300

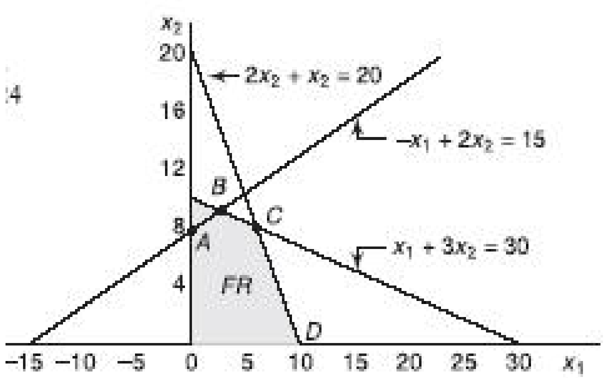
Example 5.
Solve the following LPP by graphical method
Maximize Z = 10X1 + 8X2
Subject to constraints
2X1 + X2 ≤ 20
X1 + 3X2 ≤ 30
X1 - 2X2 ≥ -15
X1 X2 ≥ 0

Solution:
The first constraint 2X1 + X2 ≤ 20 can be represented as follows.
We set 2X1 + X2 = 20
When X1 = 0 in the above constraint, we get,
2 x 0 + X2 = 20
X2 = 20
Similarly when X2 = 0 in the above constraint, we get,

2X1 + 0 = 20
X1 = 20/2 = 10

The second constraint X1 + 3X2 ≤ 30 can be represented as follows,
We set X1 + 3X2 = 30
When X1 = 0 in the above constraint, we get,
0 + 3X2 = 30
X2 = 30/3 = 10
Similarly when X2 = 0 in the above constraint, we get,
X1 + 3 x 0 = 30
X1 = 30

The third constraint X1 - 2X2 ≥ -15 can be represented as follows,
We set X1 - 2X2 = -15
When X1 = 0 in the above constraint, we get,
0 - 2X2 = -15
X2 = -15/2 = 7.5
Similarly when X2 = 0 in the above constraint, we get,
X1 – 2 x 0 = -15
X1 = -15



Point	X1	X2	Z = 10X1 + 8X2
0	0	0	0
A	0	7.5	Z = 10 x 0 + 8 x 7.5 = 60
B	3	9	Z = 10 x 3 + 8 x 9 = 102
C	6	8	Z = 10 x 6 + 8 x 8 = 124* Minimum
D	10	0	Z = 10 x 10 + 8 x 0 = 100

The Maximum profit is at point C
When X1 = 6 and X2 = 8
Z = 124

Duality in Linear Programming

Duality in Linear Programming For every LPP there is a unique LPP associated with it involving the same data and closely related optimal solution. The original problem is then called primal problem while the other is called its Dual problem

Let the primal problem be

Maximize $Z = C_1 X_1 + C_2 X_2 + \ldots\ldots + C_nX_n$

Subject to constraints,

$a_{11}X_1 + a_{12}X_2 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{1n}X_n \leq, b_1$

$a_{21}X_1 + a_{22}X_2 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{2n}X_n \leq, b_2$

$a_{31}X_1 + a_{32}X_2 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{3n}X_n \leq, b_3$

$a_{m1} X_1 + a_{m2}X_2 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{mn}X_n \leq, b_m$

and $X_1, X_2 \ldots. X_n \geq 0$

Let the primal problem be

Maximize $Z = C_1 X_1 + C_2 X_2 + \ldots\ldots + C_nX_n$

Subject to constraints,

$a_{11}X_1 + a_{12}X_2 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{1n}X_n \leq, b_1$

$a_{21}X_1 + a_{22}X_2 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{2n}X_n \leq, b_2$

$a_{31}X_1 + a_{32}X_2 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{3n}X_n \leq, b_3$

$a_{m1} X_1 + a_{m2}X_2 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{mn}X_n \leq, b_m$

and $X_1, X_2 \ldots. X_n \geq 0$

Then its Dual is

Minimize $G = b_1W_1 + b_2W_2 + b_3W_3 + \ldots\ldots. + b_mW_m$

Subject to constraints,

$a_{11}W_1 + a_{21}W_2 + a_{31}W_3 + \ldots\ldots\ldots\ldots\ldots + a_{m1}W_m \geq C_1$

$a_{12}W_1 + a_{22}W_2 + a_{32}W_3 + \ldots\ldots\ldots\ldots\ldots + a_{m2}W_m \geq C_2$

$a_{13}W_1 + a_{23}W_2 + a_{33}W_3 + \ldots\ldots\ldots\ldots\ldots + a_{m3}W_m \geq C_3$

$a_{1n}W_1 + a_{2n}W_2 + a_{3n}W_3 + \ldots\ldots\ldots\ldots\ldots + a_{mn}W_m \geq C_n$

$W_1, W_2, W_3 \ldots\ldots W_m \geq 0$

Example.1

Write the Dual of the following LPP

Min $Z = 2X_2 + 5X_3$

$X_1 + X_2 \geq 2$

$2X_1 + X_2 + 6X_3 \leq 6$

$X_1 - X_2 + 3X_3 = 4$

and $X_1, X_2, X_3 \geq 0$

Rearrange the constraints into a standard form, we get

Min $Z = 0X_1 + 2X_2 + 5X_3$

Subject to constraints,

$X_1 + X_2 + 0X_3 \geq 2$

$-2X_1 - X_2 - 6X_3 \geq -6$

$X1- X2 +3X3 \geq 4$
$-X1 + X2 -3X3 \geq -4$
and $X1, X2, X3 \geq 0$

The Dual of the above primal is as follows
Max.G = $2W1 -6W2+ 4W3 – 4W4$
Subject to constraints,
$W 1 -2W2 + W3 –W4 \leq 0$
$W 1 - W2 - W3 + W4 \leq 2$
$0W 1 - 6W2 + 3W3- 3W4 \leq 5$
$W 1, W2, W3, W4 \geq 0$

Max.G = $2W1 -6W2+ 4(W3 – W4)$
Subject to constraints,
$W 1 -2W2 + (W3 –W4) \leq 0$
$W 1 - W2 - W3 + W4 \leq 2$
$0W 1 - 6W2 – 3(W3- W4) \leq 5$
$W 1, W2, W3, W4 \geq 0$

Max.G = $2W1 -6W2+ 4W5$
Subject to constraints,
$W 1 -2W2 + W5 \leq 0$
$W 1 - W2 – W5 \leq 2$
$0W 1 - 6W2 – 3W5 \leq 5$
$W 1, W2, \geq 0$, W5 is unrestricted in sign

Example.2
Write the Dual of the following LPP
Min Z = $4X1 + 5X2- 3X3$
Subject to constraints,
$X1+ X2 + X3 = 22$
$3X1+ 5X2 - 2X3 \leq 65$
$X1+ 7X2 +4X3 \geq 120$
$X1 , X2 \geq 0$ and X3 is unrestricted

Since X3 is Unrestricted, replace X3 with (X4 - X5) and bring the problem into standard form
Min Z = $4X1 + 5X2- 3(X4 - X5)$
Subject to constraints,
$X1+ X2 + (X4 - X5) \geq 22$
$-X1- X2 - (X4 - X5) \geq - 22$
$-3X1- 5X2 + 2(X4 - X5) \geq -65$
$X1+ 7X2 +4(X4 - X5) \geq 120$
$X1 , X2 , X4 ,X5 \geq 0$

The Dual of the above primal is as follows

Max.G = 22(W1 -W2)- 65W3 + 120W4
Subject to constraints,
W 1 -W2 - 3W3 +W4$\leq$ 4
W 1 - W2 - 5W3 + 7W4 $\leq$ 5
W 1 - W2 + 2W3+ 4W4 $\leq$ -3
-W 1 + W2 - 2W3- 4W4 $\leq$ 3
W 1, W2, W3,W4 $\geq$ 0

Max.G = 22W5 - 65W3 + 120W4
Subject to constraints,
W 5- 3W3 +W4$\leq$ 4
W5 -5W3 + 7W4 $\leq$ 5
W 1 - W2 + 2W3+ 4W4 $\leq$ -3
-W 1 + W2 - 2W3- 4W4 $\leq$ 3
W 1, W2, W3,W4 $\geq$ 0