# Molecular Spectroscopy



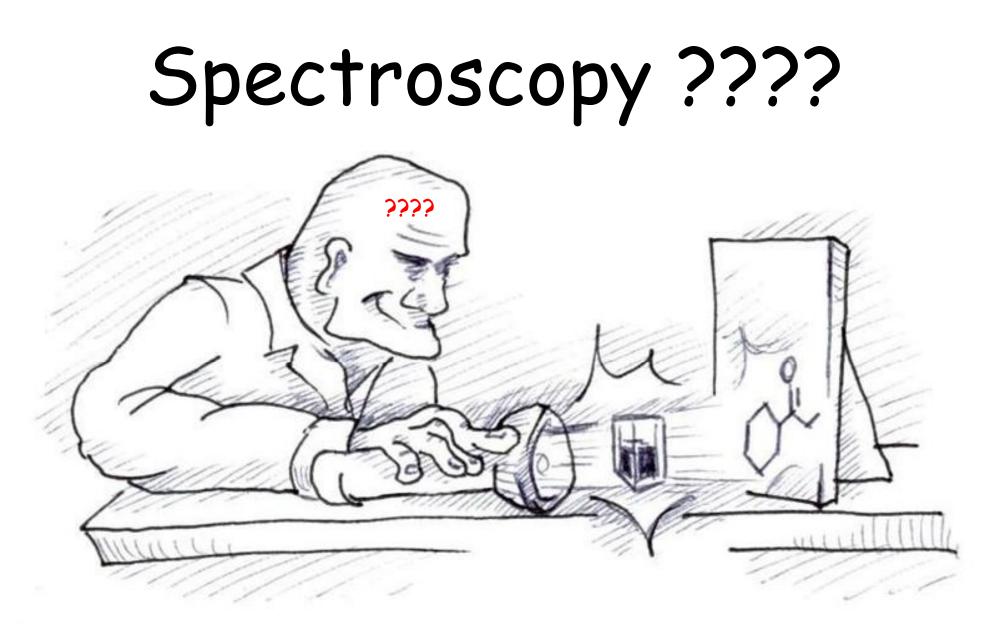
By

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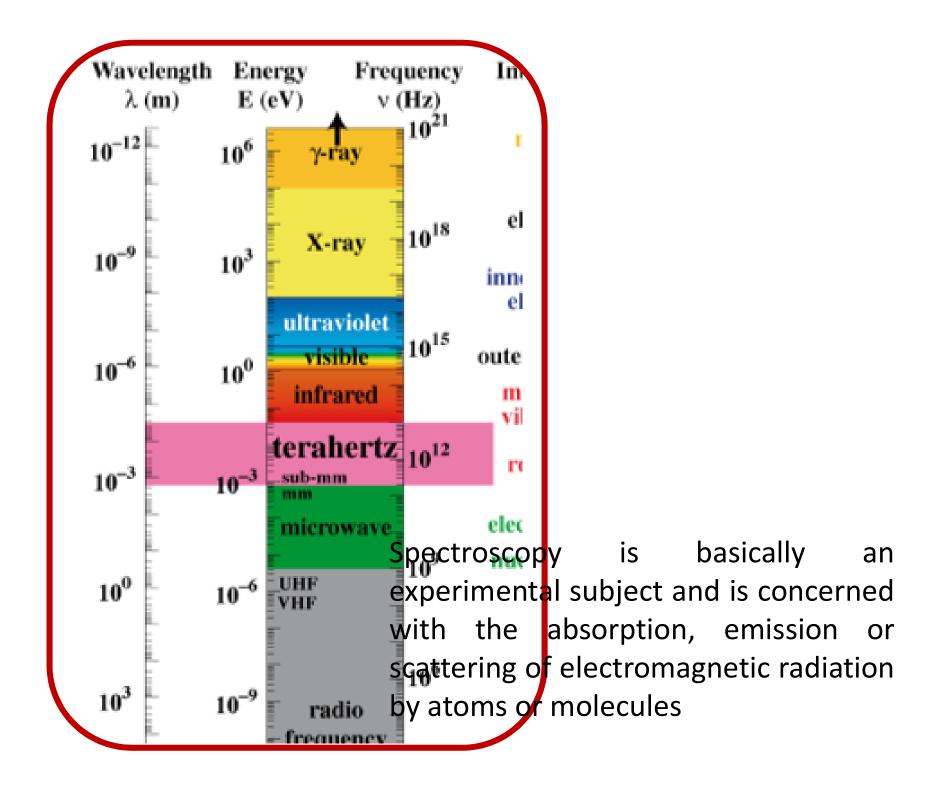
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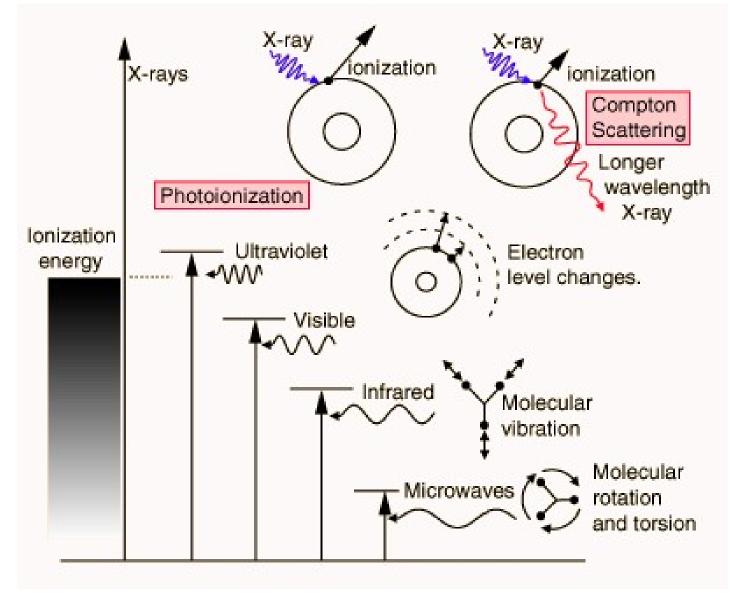
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Interaction of electromagnetic radiations with matter



### The interaction of radiation with matter



### **Different types of Energy**

Connecting macroscopic thermodynamics to a molecular understanding requires that we understand how energy is distributed on a molecular level.

**ATOMS:** <u>*The electrons*</u>: Electronic energy. Increase the energy of one (or more) electrons in the atom.

<u>Nuclear motion</u>: Translational energy. The atom can move around (translate) in space.

#### **MOLECULES:**

<u>The electrons</u>: Electronic energy. Increase the energy of one (or more) electrons in the molecule.

#### Nuclear motion:

Translational energy. The entire molecule can translate in space. Vibrational energy. The nuclei can move relative to one another.

Rotational energy. The entire molecule can rotate in space.

### **Energy is quantized**



Planck suggests that radiation (light, energy) can only come in quantized packets that are of size *hv*.

Planck, 1900 Energy (J) 
$$E = h v$$
 Frequency (s<sup>-1</sup>)

Note that we can specify the energy by specifying *any one* of the following:

1. The frequency, n (units: Hz or s<sup>-1</sup>):

$$E = hv$$

2. The wavelength,  $\lambda$ , (units: m or cm or mm): Recall:  $v\lambda = c$ 

$$E = \frac{hc}{\lambda}$$

3. The wavenumber, 
$$\widetilde{\mathcal{U}}$$
 (units: cm<sup>-1</sup> or m<sup>-1</sup>)  
Recall:  $\widetilde{\upsilon} = \frac{1}{\lambda}$ 

$$E = hc\widetilde{\upsilon}$$

#### **Absorption of Electromagnetic Radiation - The Coupling Mechanism**

An electromagnetic wave is an oscillating electrical field and interacts only with molecules that can undergo a change in **dipole moment**.

The oscillating dipole can be provided by the rotation of a permanent dipole like for example **HCI**. This type of interaction leads to microwave spectra

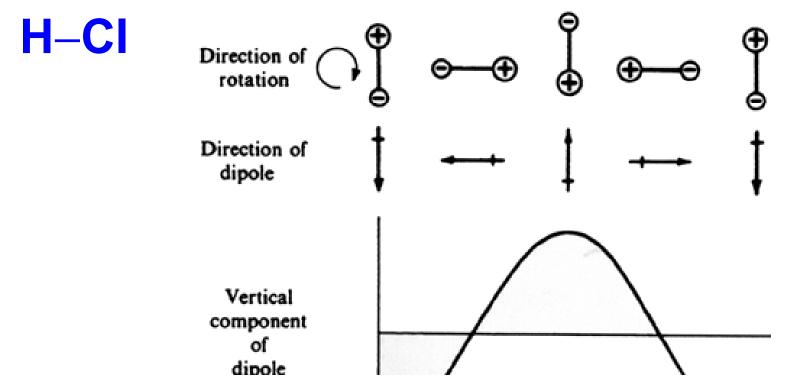
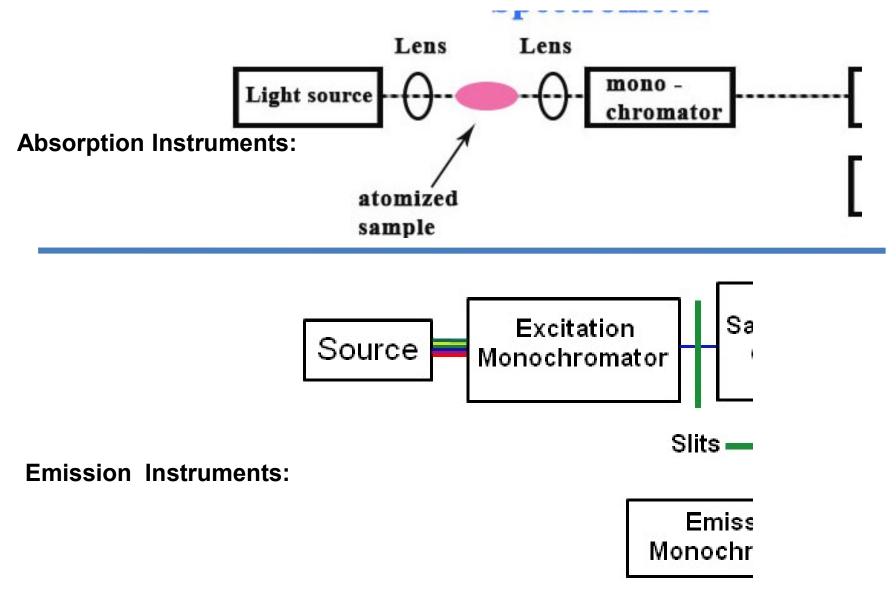


Fig. The rotation of a polar diatomic molecule, showing the fluctuation in the dipole moment measured in a particular direction

#### **Basic Elements of Practical Spectroscopy**



#### **Resolving Power**

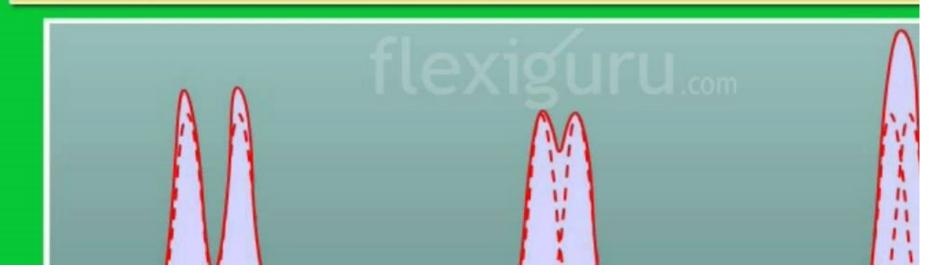
The ability of an optical instrument to produce separate images of two objects very c called resolving power.

Rayleigh criterion: When the central maximum in the diffraction pattern of one point



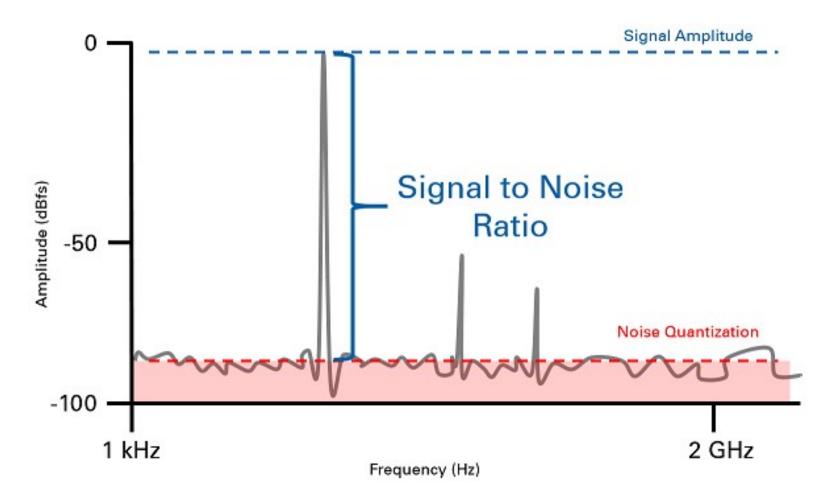


said to have been resolved by the optical instrument.



#### Signal -- to-Noise Ratio

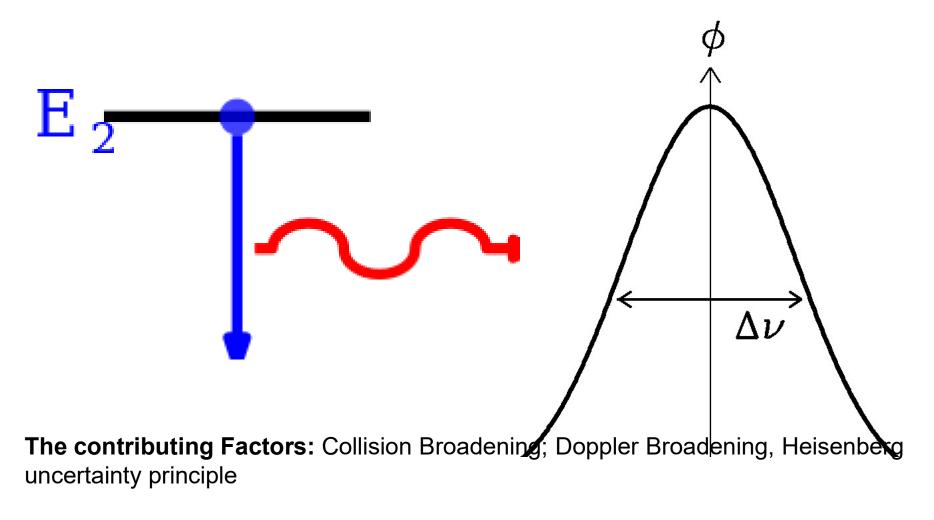
Electronic amplification to magnify signal The recorded spectrum has a background of random fluctuations by the source or detector or generated by amplifying equipments



#### Width of Spectral Lines

Range of Frequencies rather than a single frequency and every molecules are having natural line width

>Natural line width (inherent minimum width in atomic and molecular transitions)



### **Collision Broadening**

ntensity

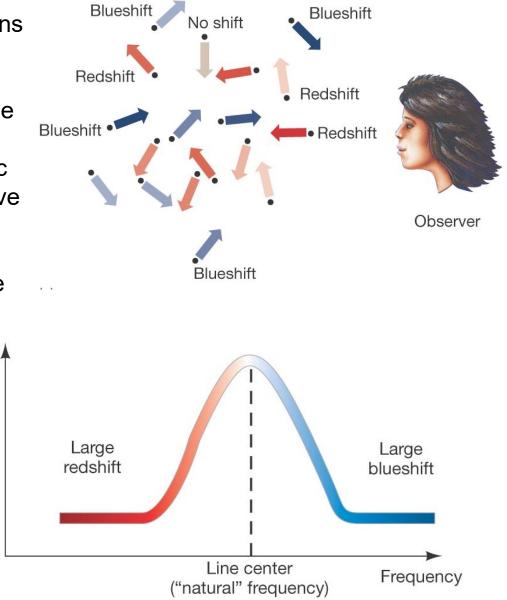
In gas and Liquid: Due to collision of an atom with other atoms, ions, free electrons or the walls of the container.

In Solids Due to the interaction of with the phonons of the lattice It leads to the change of relative phase between atomic dipole moment and that of a incident wave Collision

In solid particles are more limited: Hence solid phase spectra's are more sharper

Spectral-line broadenir

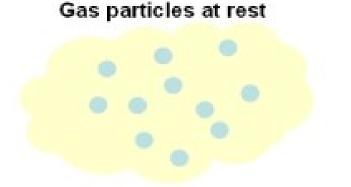
broadening



### **Doppler Broadening**

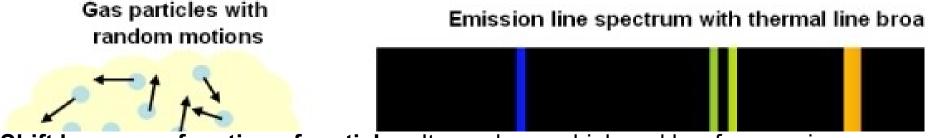
**Doppler broadening** is the broadening of spectral lines due to the **Doppler effect** caused by a distribution of velocities of atoms or molecules

**Thermal Doppler broadening:** at KT particles are not at rest position having different velocities



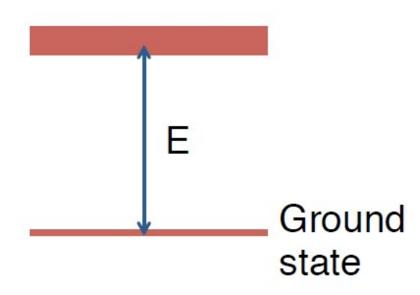
Emission line spectrum with narrow li





Shift because of motion of particles: Its random so high and low frequencies

#### **Heisenberg Uncertainty Principle**

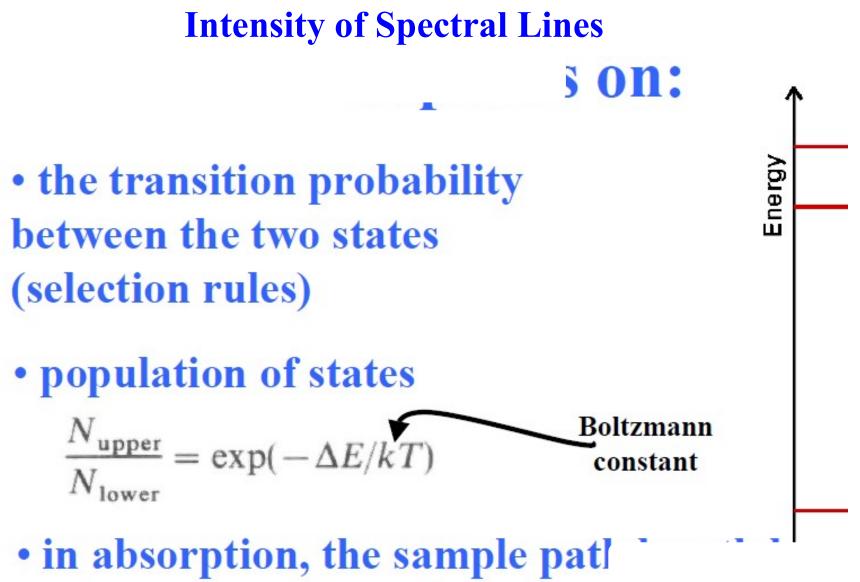


Consider excited state E above the ground sta

Electrons in excited sta there for average time decaying to ground sta

Uncertainty principle: energy of a level is uncertainty amount  $\Delta E$  given by: If  $\delta t$ = infinite then  $\delta E$ = 0

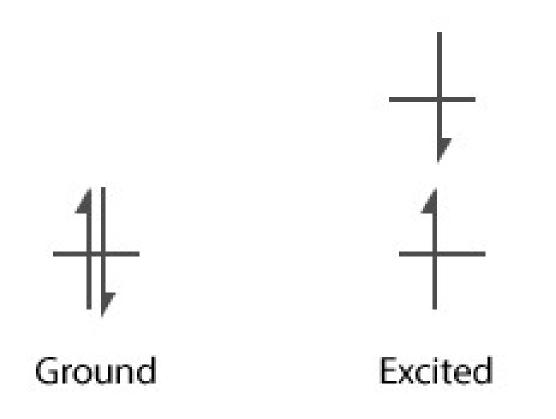
$$\Delta E \Delta t \approx \frac{h}{2\pi}$$



and concentratio

#### **Transition Probability (Selection Rule)**

Selection Rules determine the probability of the transition



## **Selection Rules**

- During the transition, there must be a change in the dipole mome molecule:
  - if there is a large change, the light / molecule interaction is st photons are absorbed:

large area or intense bands  $\rightarrow$  intense colour

if there is a small change, the light / molecule interaction is w photons are absorbed:

low area or weak bands → weak colour

If there is no change, there is no interaction and no photons:

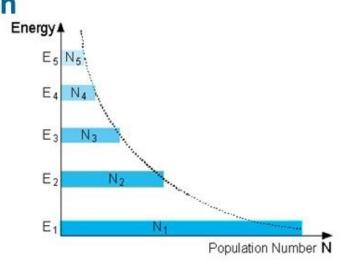
Population of States (Boltzmann Distribution) Relative Population  $(N_2/N_1)$ 

The relative population  $(N_2/N_1)$  of two energy levels  $E_2$  compared to  $E_1$  is:

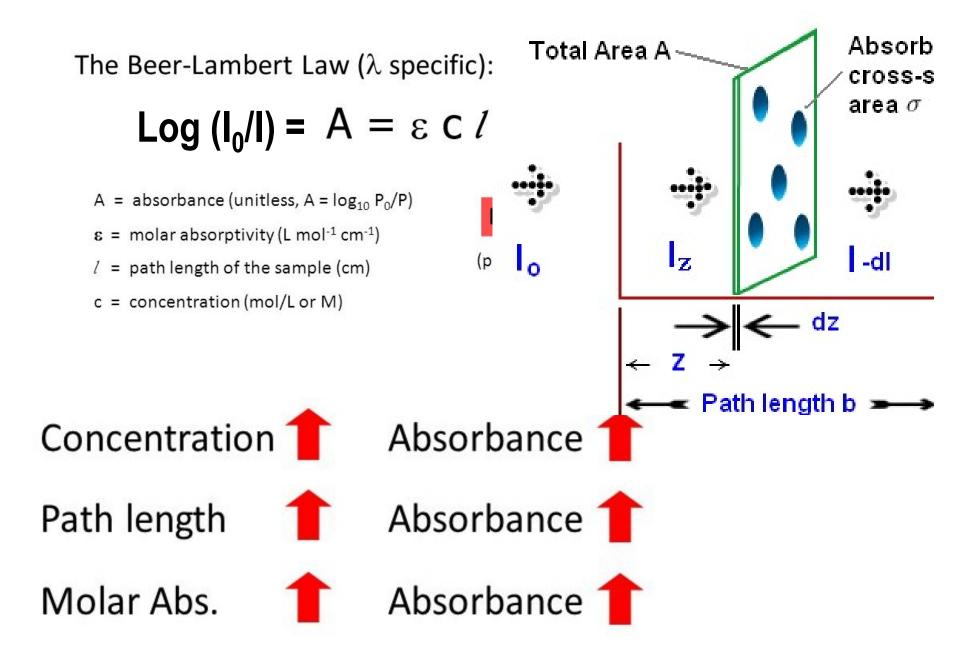
 $N_2/N_1 = \text{const}^* \exp\left(-\frac{E_2/kT}{k}\right)/\frac{1}{kt} \exp\left(-\frac{E_2-E_1}{kt}\right)$ 

The proportionality constant (const) is cancelled by division of the two population numbers.

The Figure below shows the population of each energy level at thermal equilibrium.



#### Path Length of the Sample (Molar Absorption Coefficient)



## **Microwave interactions**

Small number of available states, almost transparent.

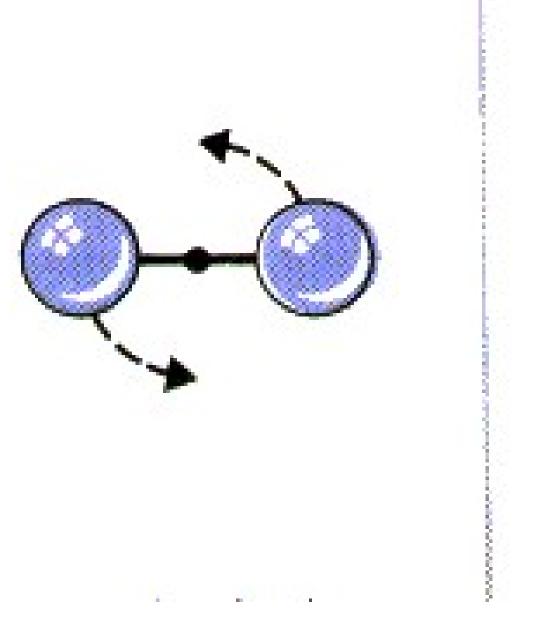
Microwaves rotate molecules

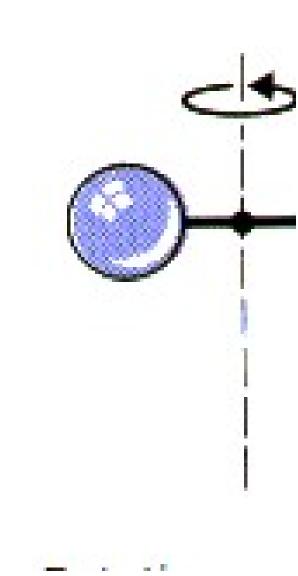


• Quantum energy of microwave photons (0.00001-0.001 eV) matches the ranges of energies separating quantum states of molecular rotations and torsion

 Note that *rotational motion of molecules is quantized*, like electronic and vibrational transitions → associated absorption/emission lines The electric field of an electromagnetic wave exerts a torque on an electric dipole.

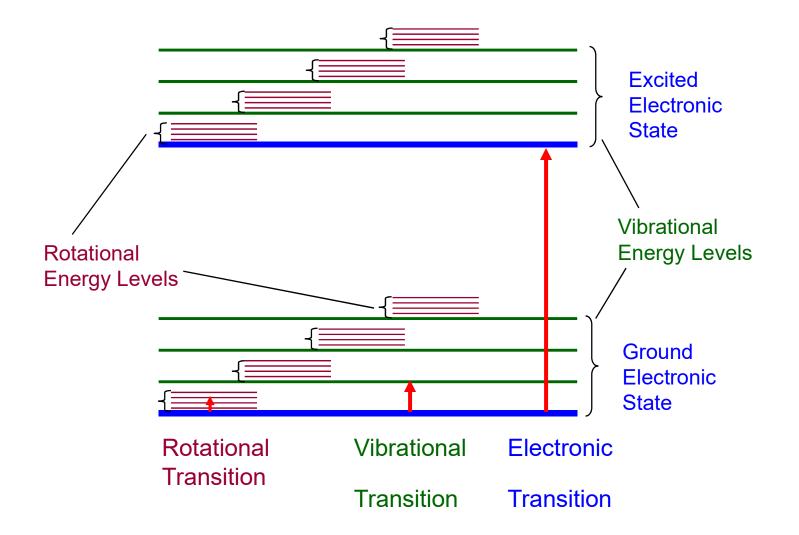
• Absorption of microwave radiation causes heating due to increased molecular rotational activity





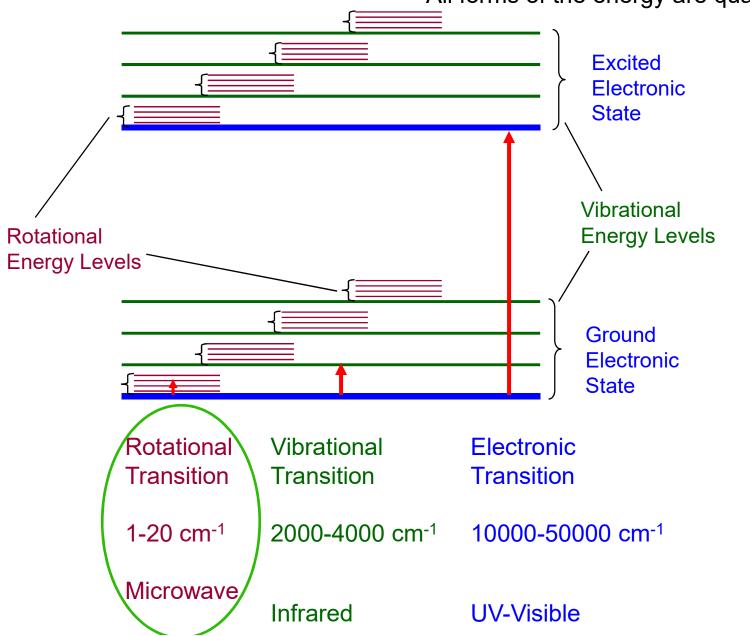
## **Molecular Energy Levels**

Radiation can be absorbed or emitted if the molecule changes any of its energy states



## **Molecular Energy Levels**

All forms of the energy are quantized



### **Rigid Molecules**

### **Non-Rigid Molecules**

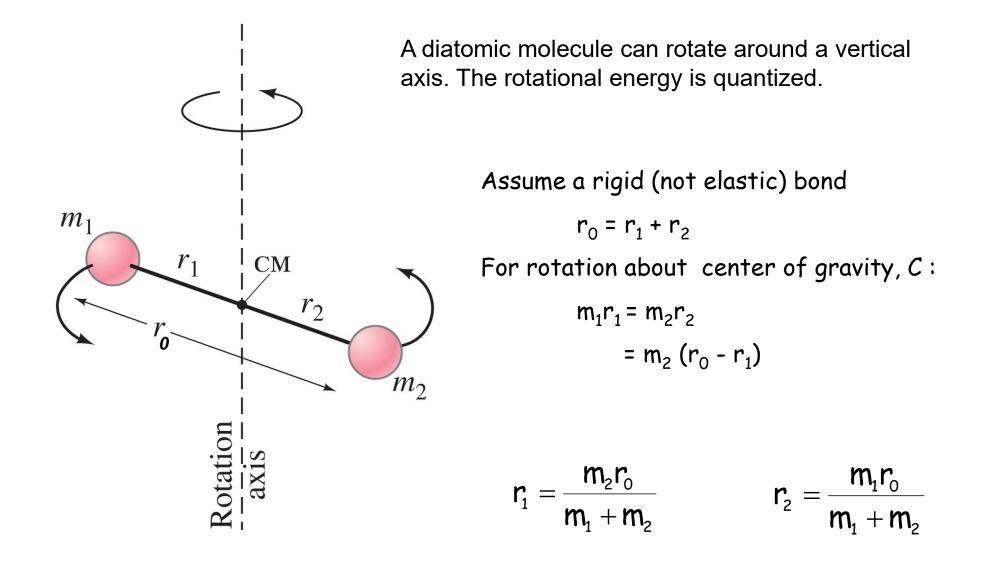
## **Rotational Spectra**

### **Diatomic Molecules**

**Polyatomic Molecules** 

### **Rigid Diatomic Molecules**

For simplicity, we can consider only rotational motion of rigid diatomic molecule



## **RIGID ROTOR**

Moment of inertia about C:

$$I_{c} = m_{1}r_{1}^{2} + m_{2}r_{2}^{2} = m_{2}r_{2}r_{1} + m_{1}r_{1}r_{2} = r_{1}r_{2} (m_{1} + m_{2})$$

$$\Rightarrow \mathbf{I} = \frac{m_1 m_2}{m_1 + m_2} r_0^2 = \mu r_0^2 \quad \mu = \text{reduced mass}, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

A diatomic molecule can rotate around a vertical axis. The rotational energy is quantized. By using the Schrödinger equation, the rotational energy levels allowed to the rigid diatomic molecule are given by,

$$E_J = \frac{h^2}{8\pi^2 I} J(J+1)$$
 joules where  $J = 0, 1, 2, ...$ 

J = Rotational quantum number (J = 0, 1, 2, ...)

I = Moment of inertia = 
$$mr^2$$

$$\mu$$
 = reduced mass = m<sub>1</sub>m<sub>2</sub> / (m<sub>1</sub> + m<sub>2</sub>)

r = internuclear distance

#### **Rotational Spectra of Rigid Diatomic molecule**

Rotational Energy Levels for rigid rotor:

$$E_J = \frac{h^2}{8\pi^2 I} J(J+1)$$
 joules where  $J = 0, 1, 2, ...$ 

$$\varepsilon_J = \frac{E_J}{hc} = \frac{h}{8\pi^2 Ic} J(J+1) \text{ cm}^{-1} \qquad (J=0, 1, 2, \ldots)$$

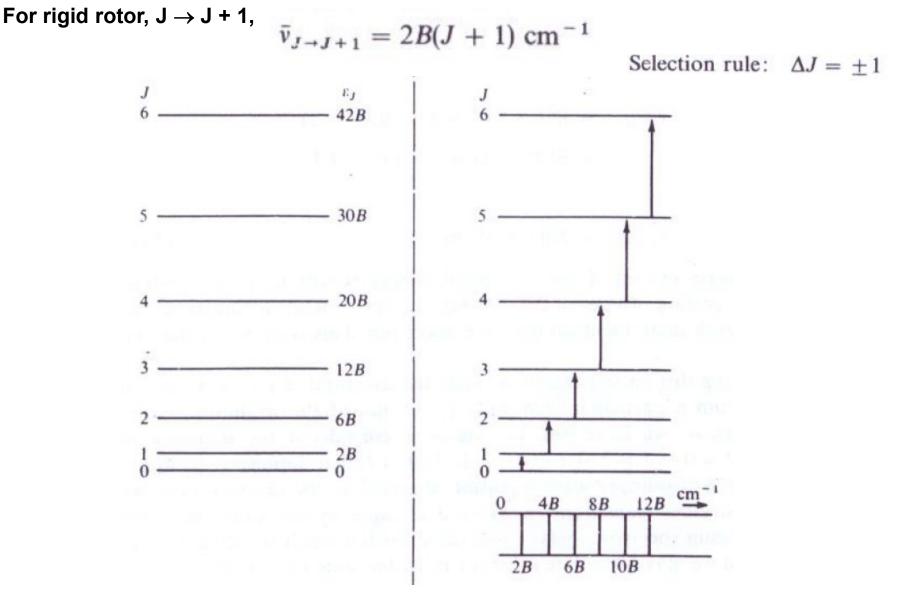
$$\varepsilon_J = BJ(J+1) \text{ cm}^{-1}$$
 (J = 0, 1, 2, ...)

where B, the rotational constant, is given by

$$B = \frac{h}{8\pi^2 I_B c} \quad \text{cm}^{-1}$$

Where 
$$I = \frac{m_1 m_2}{m_1 + m_2} r_0^2 = \mu r_0^2$$

#### **Rotational Spectra of Rigid Diatomic molecule**

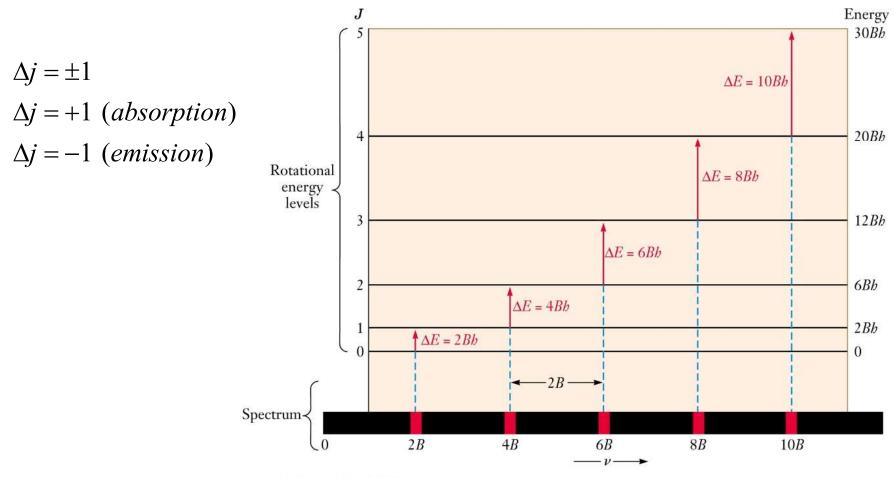


The allowed rotational energy levels of a rigid diatomic molecule

Allowed transitions between the energy levels of a rigid diatomic molecule and the spectrum

#### **Rotational Spectra of Rigid Rotor**

**Selection Rule**: Apart from Specific rule,  $\Delta J = \pm 1$ , Gross rule- the molecule should have a permanent electric dipole moment,  $\mu$ . Thus, homonuclear diatomic molecules do not have a pure rotational spectrum. Heteronuclear diatomic molecules do have rotational spectra.



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**Appearance of rotational spectrum** We can calculate the energy corresponding to rotational transitions

$$\Delta$$
 E=EJ'-EJ for  $\Delta J = J_{final} - J_{initial} = +1$ 

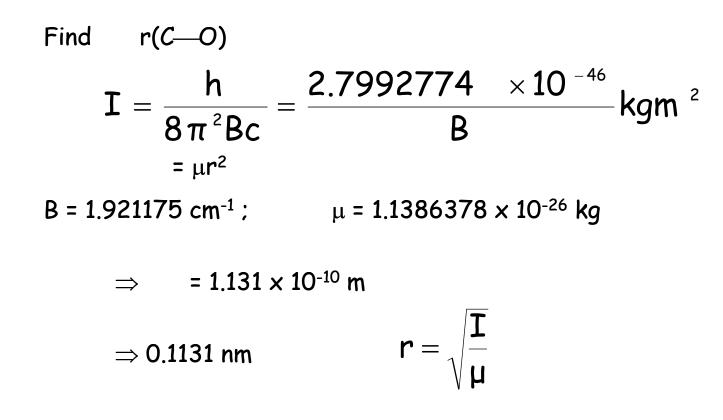
Or generally:  

$$J \rightarrow J + 1$$
 $v = B(J+1)(J+2) - BJ(J+1)$ 
 $= 2B(J+1) \text{ cm}^{-1}$ 

Microwave absorption lines should appear at

 $\begin{array}{ccc} J=0 & \rightarrow J=1: & \overline{v} & = 2B-0 = 2B \ cm^{-1} \\ J=1 & \rightarrow J=2: & = & = 4B \ cm^{-1} \end{array}$ 

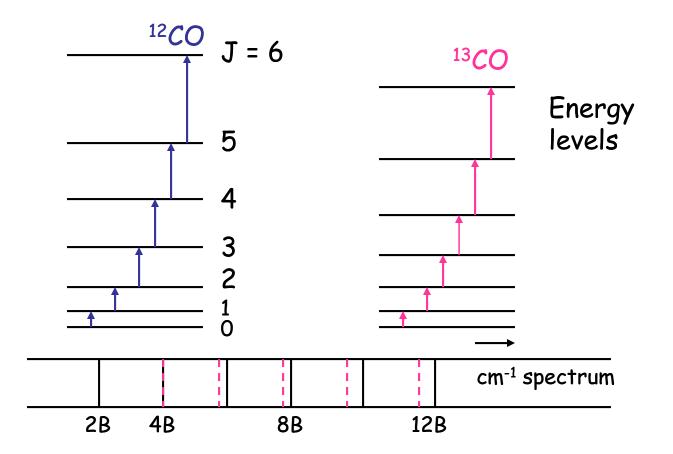
Note that the selection rule is  $\Delta J = \pm 1$ , where + applies to absorption and - to emission.



Answer: C-O bond length is 0.1131 nm.

#### Effect of isotopes

From  ${}^{12}C^{16}O \rightarrow {}^{13}C^{16}O$ , mass increases, B decreases ( $\propto 1/I$ ), so energy levels lower.



# Comparison of rotational energy levels of $^{12}\mathrm{CO}$ and $^{13}\mathrm{CO}$

Can determine:

(i) isotopic masses accurately, to within 0.02% of other methods for atoms in gaseous molecules;

(ii) isotopic abundances from the absorption relative intensities.

#### Example:

for <sup>12</sup> CO	$J=0 \rightarrow J=1$	at 3	3.84235 cm <sup>-1</sup>
for <sup>13</sup> CO		3.67	337 cm <sup>-1</sup>
Given : <sup>12</sup> C = 12.0000 ;	O = 15.99	994	amu

# Relative Intensities of rotation spectral lines

Now we understand the locations (positions) of lines in the microwave spectrum, we can see which lines are strongest.

Intensity depends upon two factors:

#### Intensity depends upon two factors:

1.Greater initial state population gives stronger spectral lines.This population depends upon temperature, T.

k = Boltzmann's constant, 1.380658 x 10<sup>-23</sup> J K<sup>-1</sup>  
(k = R/N) 
$$\frac{N_J}{N_0} \propto \exp\left(-\frac{E_J}{kT}\right) = \exp\left(-\frac{hc}{kT}\right)$$

 $\label{eq:states} \begin{array}{l} \mbox{We conclude that the population is smaller for higher J states.} \\ \mbox{$\frac{N_{_J}}{N_{_o}} \propto e \left(-\frac{1.52034v}{T}\right)$} \\ \mbox{$\frac{hc}{k} = 1.52034$ cmK$} \end{array}$ 

# 2. Intensity also depends on degeneracy of initial state.

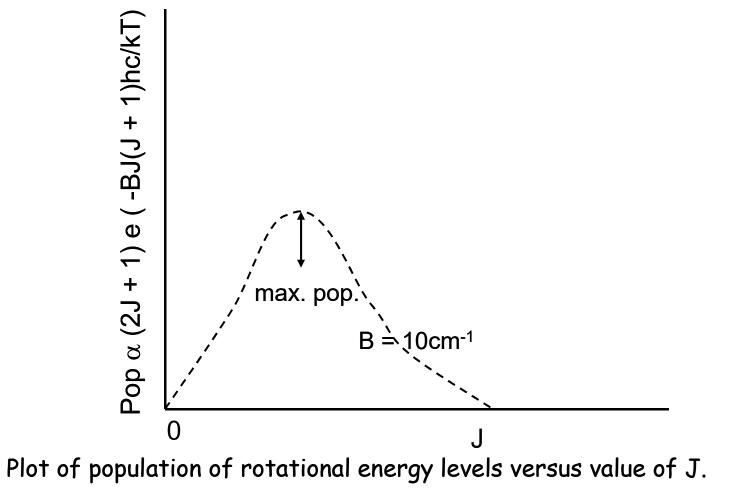
(degeneracy = existence of 2 or more energy states having exactly the same energy)

Each level J is (2J+1) degenerate

 $\Rightarrow$  population is greater for higher J states.

To summarize: Total relative population at energy  $E_J \alpha$  (2J+1) exp (- $E_J / kT$ ) & maximum population occurs at nearest integral J value to :

Look at the values of  $N_J/N_0$  in the figure, .



# **The Non-Rigid Rotor**

If the shape of the molecule is allowed to distort upon rotation, Then the restriction of the rigid rotor is lifted.

It may be expected that as the rotational energy increases the molecule will have it's bond lengthened because of the centrifugal distortion.

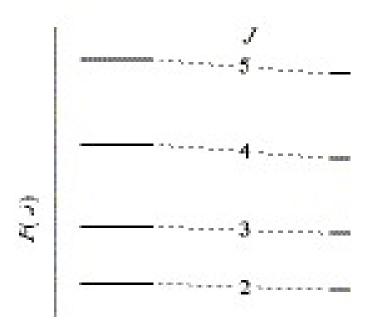
This will lead to an effective decrease in the rotational energy since the longer bond will lead to a decrease in the rotational energy.

We can refine the theory by adding a correction term, containing the centrifugal distortion constant, D, which corrects for the fact that the bond is not rigid.

$$F(J) = \tilde{B}J(J+1) - \tilde{D}J^2(J+1)^2$$

Here, *D* is the centrifugal distortion constant. For the  $J \rightarrow J + 1$  transitions, the absorption frequency is given by

$$\tilde{\nu} = F(J+1) - F(J) = 2\tilde{B}(J+1) - 4\tilde{D}(J+1)^3 \qquad J = 0, \ 1, \ 2, \ \dots$$



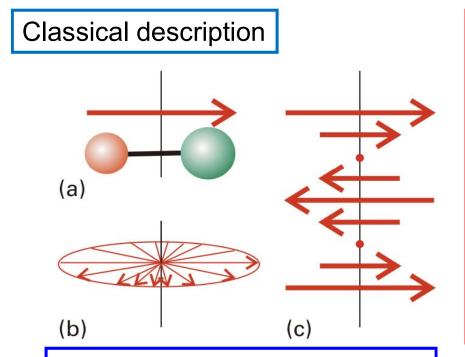
$$D = \frac{h^3}{32\pi^4 I^2 r^2 kc} cm^{-1}$$
$$= \frac{4B^3}{\overline{v}_{vib}^2}$$

where  $_{V_{vib}}^{-}$  is bond stretch wavenumber. i) can find J values of lines in a spectrum - fitting 3 lines gives 3 unknowns: J, B, D.

ii) We can estimate  $v_{vib}$  from the small correction term, D.

# **Rotational spectroscopy**

• **Gross selection rule**: The molecules must have a permanent electric dipole moment so that the molecules are polar.



To an observer, a rotating polar molecule has an electric dipole that appears to oscillate. This oscillating dipole can interact with the EM field. **Rotational-inactive molecules**: Molecules without rotational spectrum Homonuclear diatomic molecules:  $N_2$ ,  $O_2$ Symmetric linear molecules:  $CO_2$ Tetrahedral molecules:  $CH_4$ Octahedral molecules:  $SF_6$ ,  $C_6H_6$ 

**Rotational-active molecules**: Molecules with rotational spectrum Heteronuclear diatomic molecules: HCl Less symmetric polar molecules: NH<sub>3</sub>, H<sub>2</sub>O

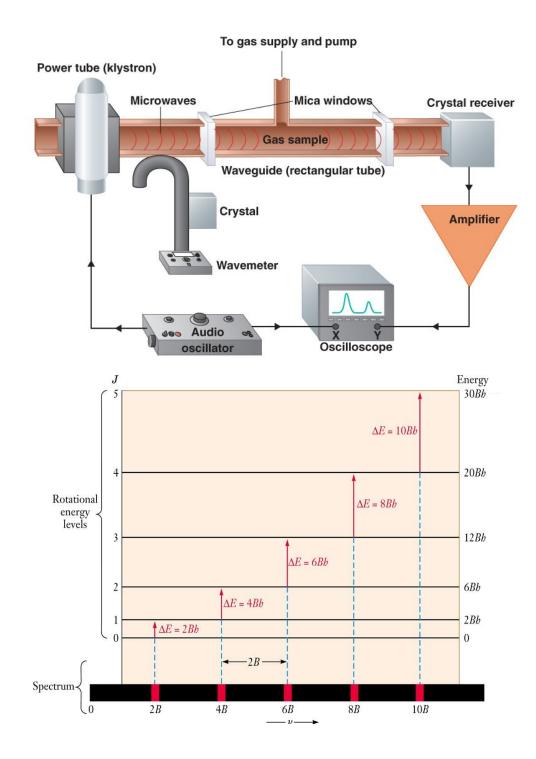
#### **Microwave Spectroscopy**

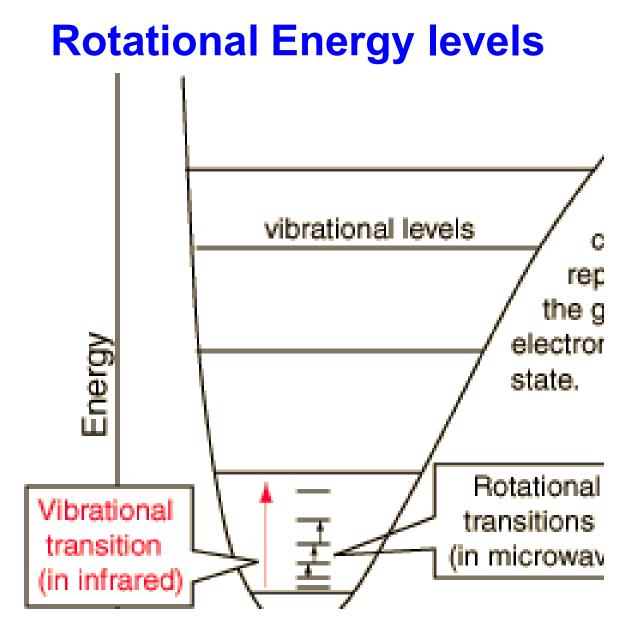
Incident electromagnetic waves can excite the rotational levels of molecules provided they have an <u>electric dipole moment</u>. The electromagnetic field exerts a <u>torque</u> on the molecule.



Homonuclear diatomic molecules (such as H<sub>2</sub>, O<sub>2</sub>, N<sub>2</sub>, Cl<sub>2</sub>) – have zero dipole (non polar) -- have zero change of dipole during the rotation – hence <u>NO</u> interaction with radiation -- hence <u>homonuclear diatomic molecules are microwave inactive</u>

Heteronuclear diatomic molecules (such as HCI, HF, CO) – have permanent dipole moment (polar compound) -- change of dipole occurs during the rotation – hence interaction with radiation takes place – Therefore, <u>heteronuclear diatomic molecules</u> <u>are microwave active.</u>

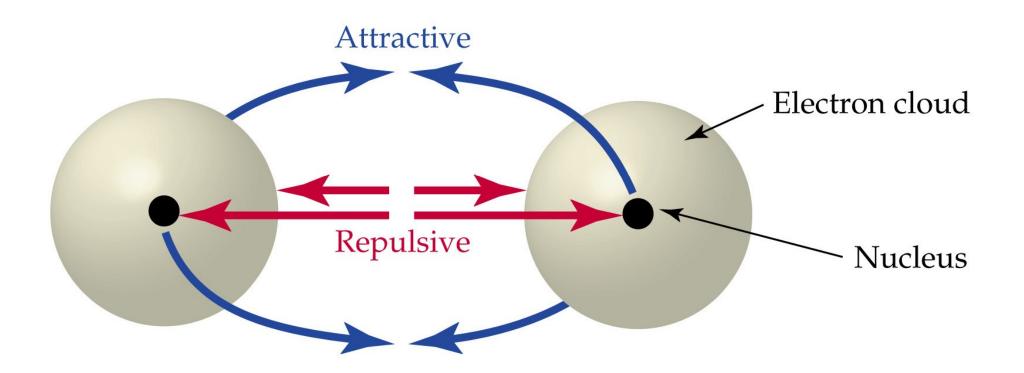




The spectra for rotational transitions of molecules is typically in the microwave region of the electromagnetic spectrum.

# Vibrational Spectroscopy

## **Chemical Bond Formation**



The two atoms settle at a mean internuclear distance such that these forces are just balanced and the total energy of the whole system is a minimum

# Vibrational Energy of Diatomic Molecules

The bonds between atoms behave like springs



• The atoms vibrate approximately like an harmonic oscillator obeying Hooke's Law:

$$F = -k(r - r_{eq})$$
 k is the force constant

$$E_{PE} = \frac{1}{2}k(r - r_{eq})^2$$

# Vibrational Energy of Diatomic Molecules

$$\omega_{\text{osc}} = \frac{1}{2\omega c} \sqrt{\frac{Hz}{2}}$$

$$\omega_{\rm osc} = \frac{1}{2\pi c} \sqrt{\frac{cm^{-1}}{2\pi c}}$$

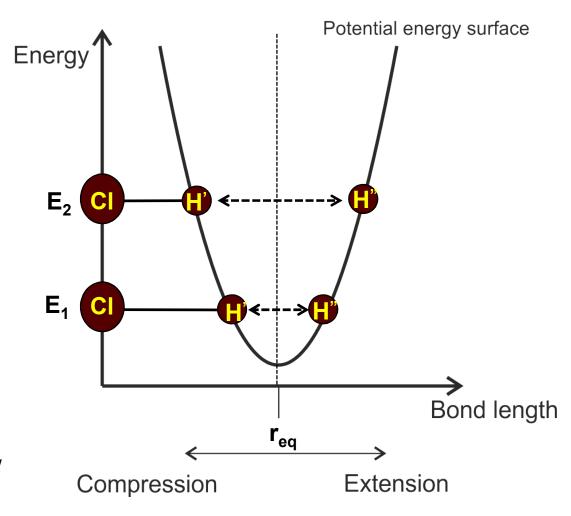
Allowed vibrational energy levels

$$\mathsf{E}_{\mathsf{vib}} = (\mathsf{v} + \frac{1}{2})\mathsf{h}\omega_0 \mathsf{J}$$

V = 0, 1, 2, ...

$$E_{v} = (v+1/2) h\omega_{osc} cm^{-1}$$

Exchange of PE and KE during vibration



#### Vibrational spectra: Harmonic oscillator model

**Oscillation Frequency,** 

$$\omega_{\rm osc.} = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}} \quad \text{Hz}$$
$$\bar{\omega}_{\rm osc.} = \frac{1}{2\pi c} \sqrt{\frac{k}{\mu}} \quad \text{cm}^{-1}$$

*k* is the force constant  $\mu$  = reduced mass,

# Vibrational frequency only dependent on the mass of the system and the force constant .

From Schrodinger equation, Vibrational energies for simple harmonic oscillator,

$$E_v = (v + \frac{1}{2})h\omega_{\text{osc.}}$$
 joules  $(v = 0, 1, 2, ...)$ 

where v is called the vibrational quantum number.

$$\varepsilon_v = \frac{E_v}{hc} = (v + \frac{1}{2})\overline{\omega}_{\text{osc.}} \quad \text{cm}^{-1}$$

#### Vibrational spectra: Harmonic oscillator model

**Oscillation Frequency**,

$$\omega_{\rm osc.} = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}} \quad {\rm Hz}$$

*k* is the force constant  $\mu$  = reduced mass,

The vibrational frequency is increasing with:

increasing force constant k ( = increasing bond strength)

decreasing atomic mass

• Example: 
$$k_{c=c} > k_{c=c} > k_{c-c}$$

# Vibrational Energy of Diatomic Molecules

Allowed vibrational energy levels

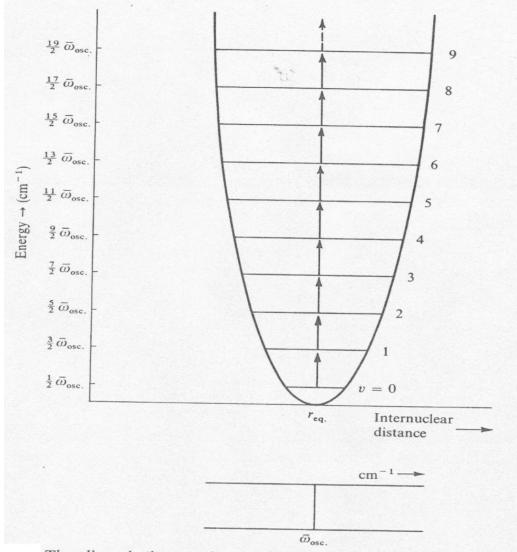
 $E_{vib} = (v + \frac{1}{2})\omega_0 \text{ cm}^{-1}$ 

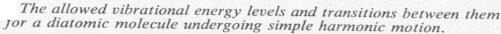
V = 0, 1, 2, ...

Allowed transitions

 $\Delta v = \pm 1$ 

Thus expect only one vibrational peak in the IR spectrum





## Vibrational Spectrum of Diatomic Molecules

Interaction between EMR and the vibrational energy of molecules can only occur if the bond is polar and a change of dipole moment occurs during oscillation.

Thus only polar bonds generate peaks in the infrared spectrum of molecules.

Thus HCl, CO and HF absorb EMR and have an IR spectrum but  $H_2$  and  $N_2$  do not.

# **Infrared Spectroscopy**

$$\omega_{\rm osc.} = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}} \quad \text{Hz}$$

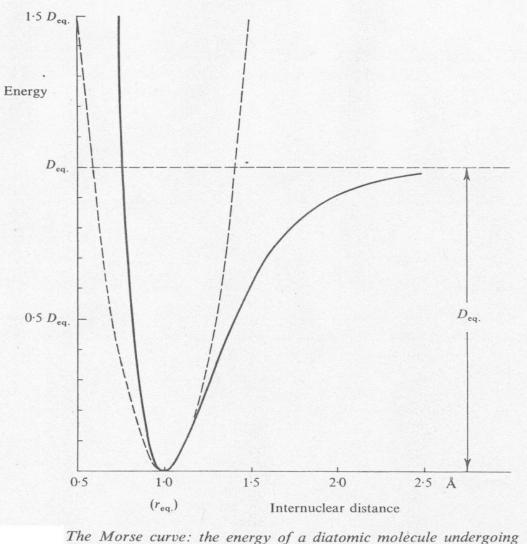
POSITION	REDUCED MASS	LIGHT ATOMS
	BOND STRENGTH (STIFFNESS)	STRONG BONDS

The vibrational frequency is increasing with:

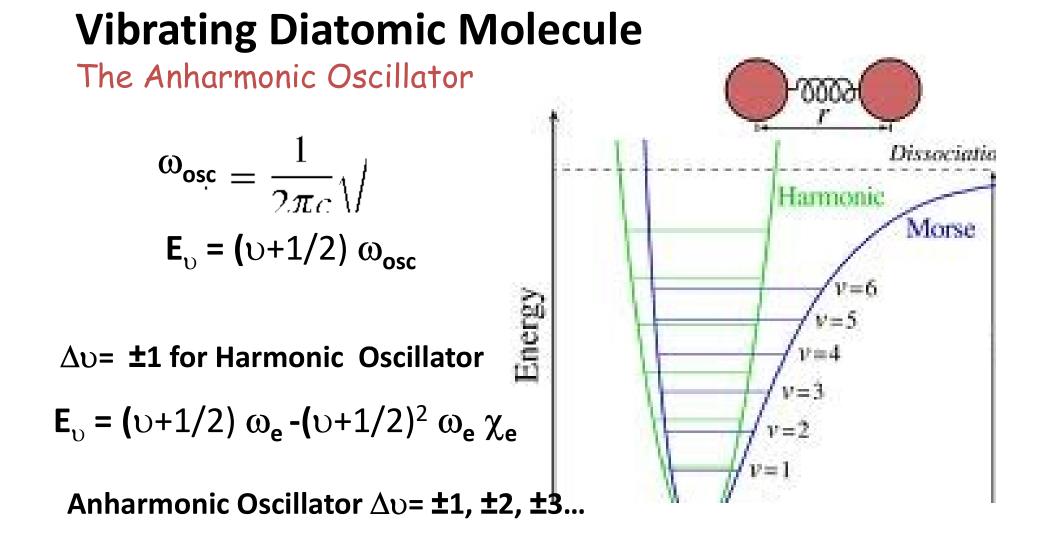
- increasing force constant *k* (= increasing bond strength)
- decreasing atomic mass

## Vibrational Energy of Diatomic Molecules The Anharmonic Oscillator

Deviations in the energy profile of a real molecule undergoing anharmonic vibration.



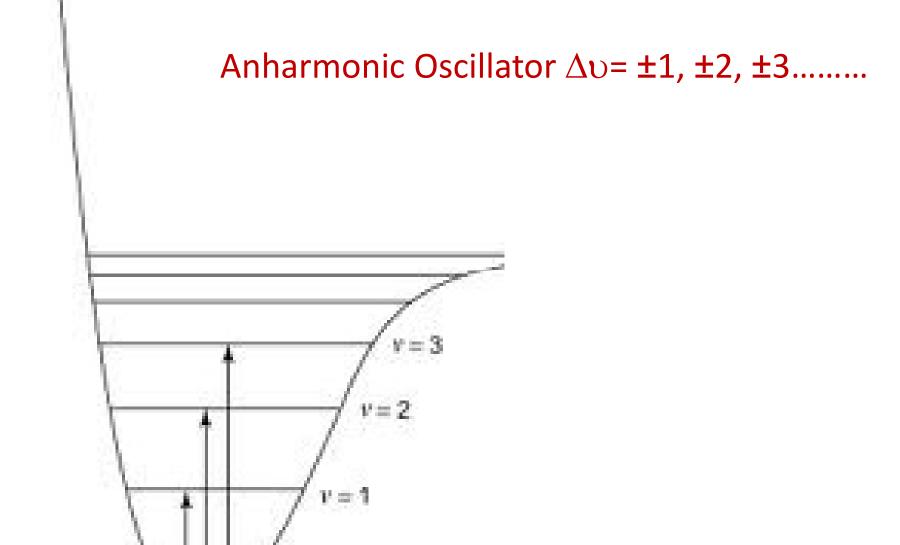
anharmonic extensions and compressions.



Real Molecules do not obey Hooks low.....bcz bonds are not elastic .....

Overtones and hot bands.....

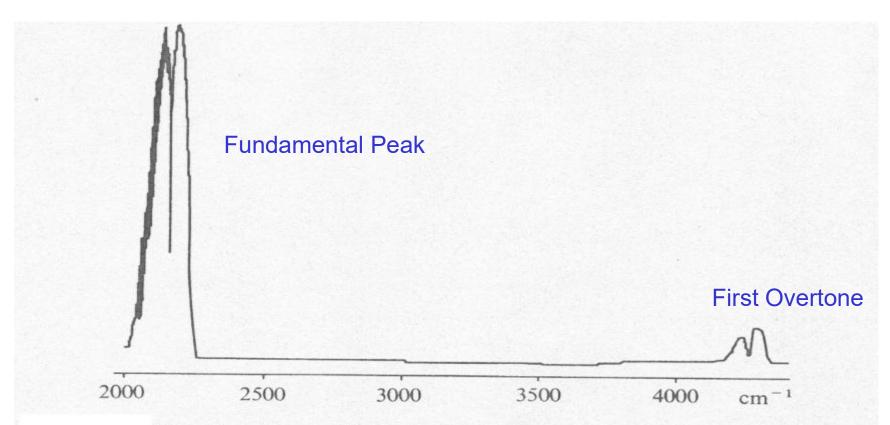
## **Anharmonic Vibrating Energy Levels**



Overtones and hot bands......

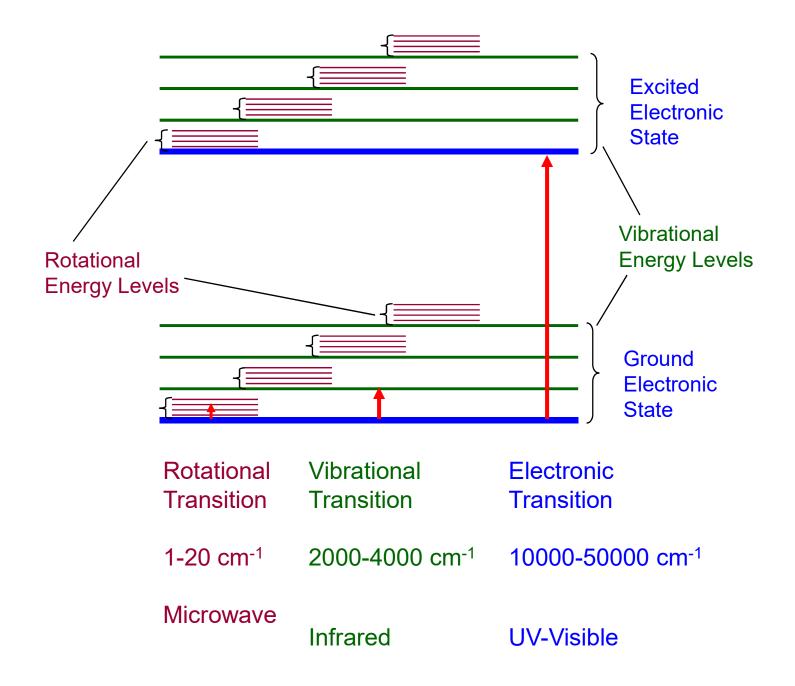
Potential energy

# IR Spectrum of Carbon Monoxide (CO)



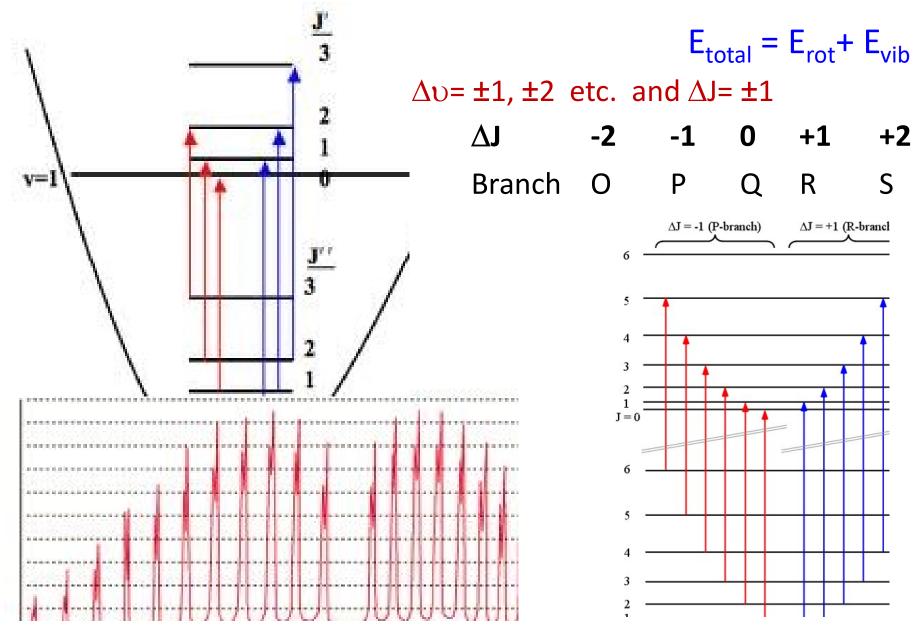
The fundamental absorption (centred at about 2143 cm<sup>-1</sup>) and the first overtone (centred at about 4260 cm<sup>-1</sup>) of carbon monoxide; the fine structure of the P branch in the fundamental is partially resolved. (Gas pressure 650 mm Hg in a 10 cm cell.)

#### **Molecular Energy Levels**



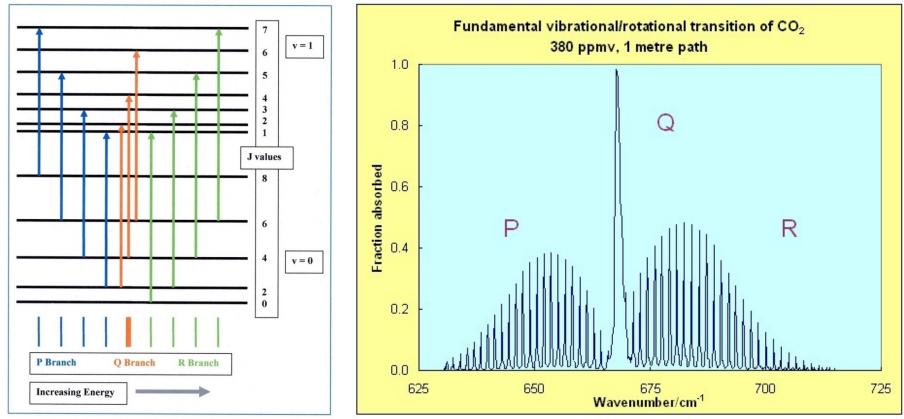
## **Diatomic Vibrating Rotator**

•Born-Oppenheimer Approximation: Energies are simply sum of separate energies



## **Interactions of Rotations and Vibrations**

- •Breakdown of Born-Oppenheimer Approximation
- •Bond-length changes with vibrational energy
- •Rotational constant (B) varies with 1/r<sup>2</sup>
- •B is smaller for upper state than lower state Energies are
- simply sum of separate energies (B<sub>1</sub><B<sub>0</sub>)  $\Delta v = \pm 1$ , and  $\Delta J = 0, \pm 1$



# Molecular Motion and Spectroscopy <u>Spectroscopy</u>

- Study of Interaction of Matter and Light (Photon)
- Molecular Spectroscopy

→ Information about molecules such as geometry and energy levels are obtained by the interaction of molecules and photons

• Molecular motions: Translation, Rotation, Vibration

 $\rightarrow$  determines the energy levels for the absorption or emission of photons

## Degrees of Freedom: Translation, Rotation, and Vibration

#### **Consider a single Ar atom moving in 3-D space:**

- Moving motion is referred to as Translation
- To analyze the translation of an Ar, we need to know position (x, y, z) and momentum (p<sub>x</sub>, p<sub>y</sub>, p<sub>z</sub>)
   Where it is Where it is headed

- Each coordinate-momentum pair [for example,  $(x,p_x)$ ] is referred to as a Degree of Freedom (DF)

- An Ar atom moving through 3-D space has three DFs

N argon atoms possesses 3N DFs: All translational DFs

## **Internal Motions**

- A collection of N atoms possesses 3N DFs
- If N atoms happen to be bonded together into an N-atom molecule, the number of DFs is still 3N.
- But, atoms in a molecule cannot translate independently of each other → No 3N translational DFs →Contribution of DFs to Internal Motions
- Two types: Rotation and Vibration

# Center of Mass (Balanced Point)

- A point mass that can represent the molecule
- Motion of the center of mass requires 3 DFs to describe it
- In general, regardless of its size or complexity, a molecule has 3 translational DFs
- Thus, (3N 3) DFs for the internal motions of rotation and vibration

## **Molecular vibrations**

*Translation* - the movement of the entire molecule while the positions of the atoms relative to each other remain fixed: <u>3 degrees of translational freedom</u>.

**Rotational transitions** – interatomic distances remain constant but the entire molecule rotates with respect to three mutually perpendicular axes: 3 rotational freedom (nonlinear), 2 rotational freedom (linear).

## **Fundamental Vibrations**

*Vibrations* – relative positions of the atoms change while the average position and orientation of the molecule remain fixed.

Degrees of freedom	linear	non-linear
Translational Rotational Vibrational	3 2 3 <i>N</i> –5	3 3 3 <i>N</i> –6
Total	3N	3 <i>N</i>

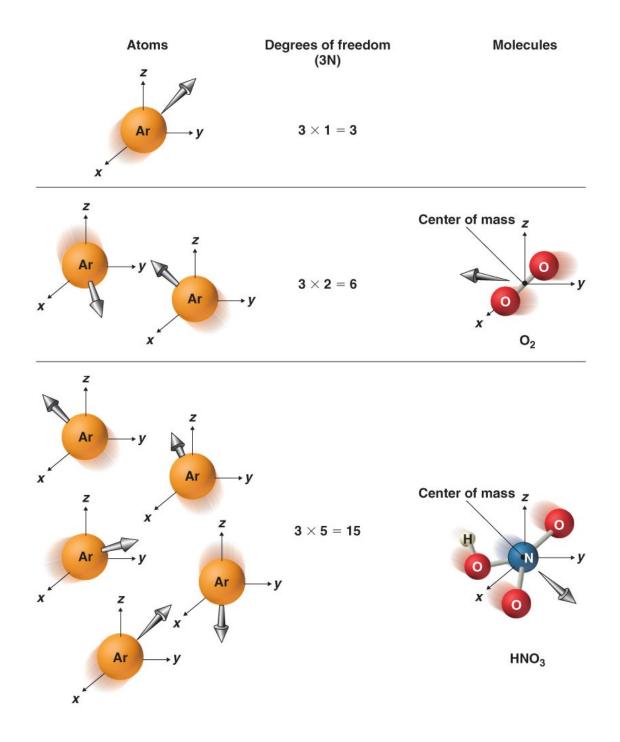
N = number of atoms in molecule

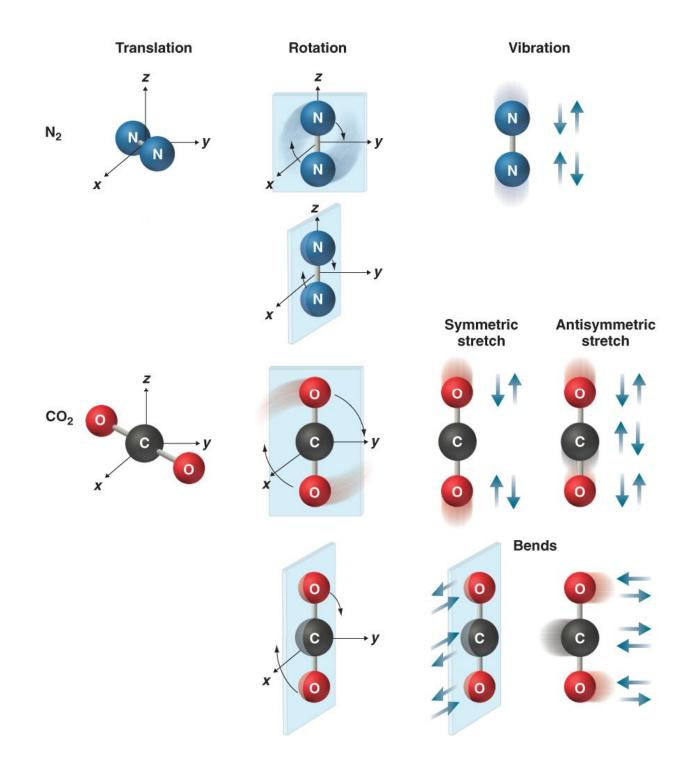
### **Molecular vibrations**

#### TABLE 8.1

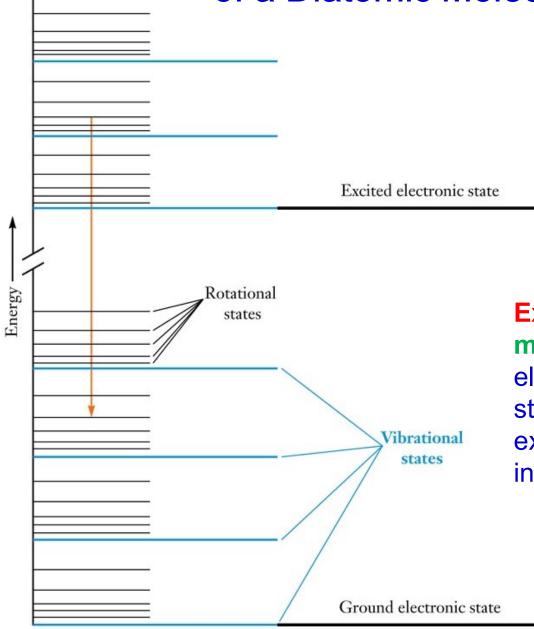
#### **Classification of degrees of freedom**

Molecule	Total	Translational	Rotational	Vibrational
<i>N</i> -atom	3 <i>N</i>	3	2 if linear	3 <i>N</i> –5 if linear
molecule			3 if nonlinear	3 <i>N</i> –6 if nonlinear
$N_2$	6	3	2	1
$CO_2$	9	3	2	4
H <sub>2</sub> O	9	3	3	3
$CH_4$	15	3	3	9

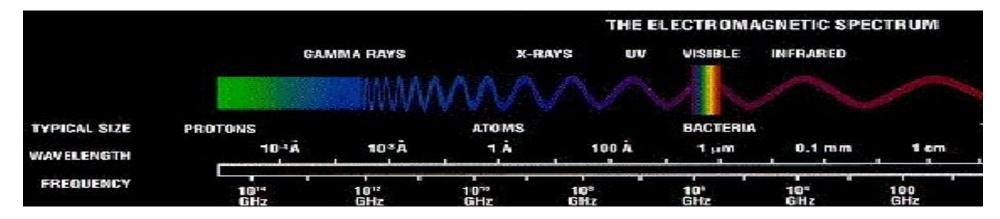


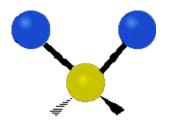


### Electronic, Vibrational, and Rotational Energy Levels of a Diatomic Molecule



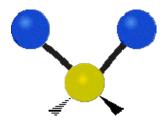
Exercise: Indicate the molecular state in which it is electronically in the ground state, vibrationally in the first excited state, and rotationally in the ground state.



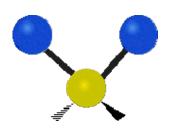


Symmetric Stretch

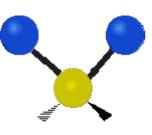
#### **Vibrational Motions**



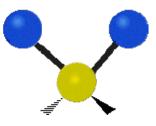
Asymmetric Stretch



Scissoring



Rocking



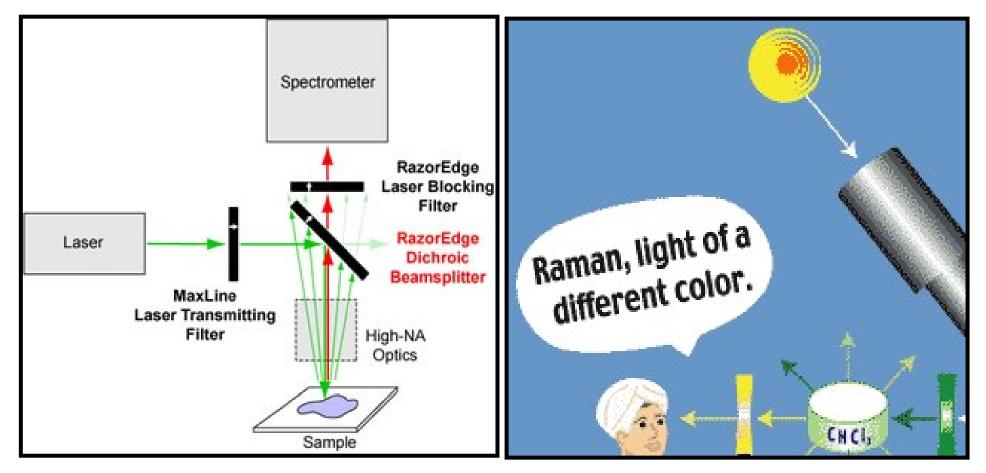
Twisting

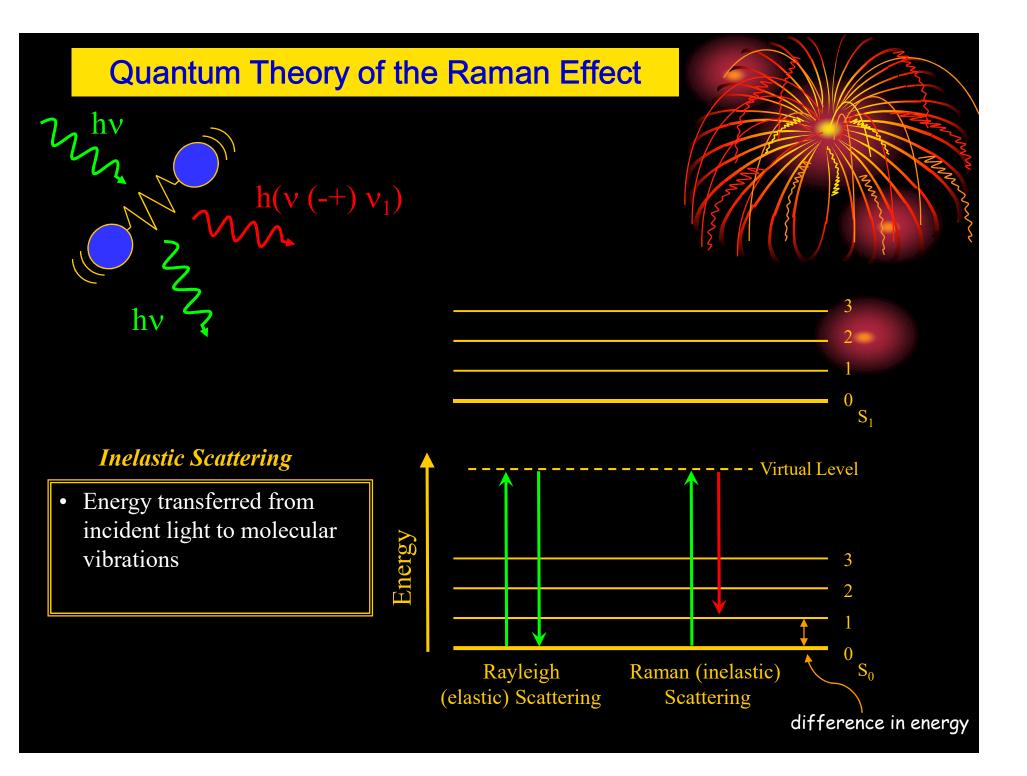
# Raman Spectroscopy

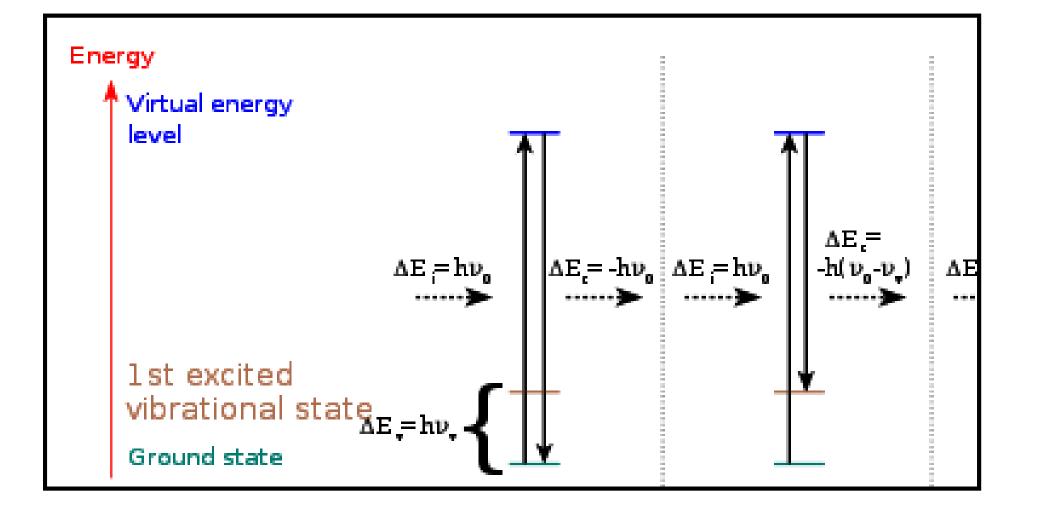
Transparent Materials...... Scattering of Light (0.01%)

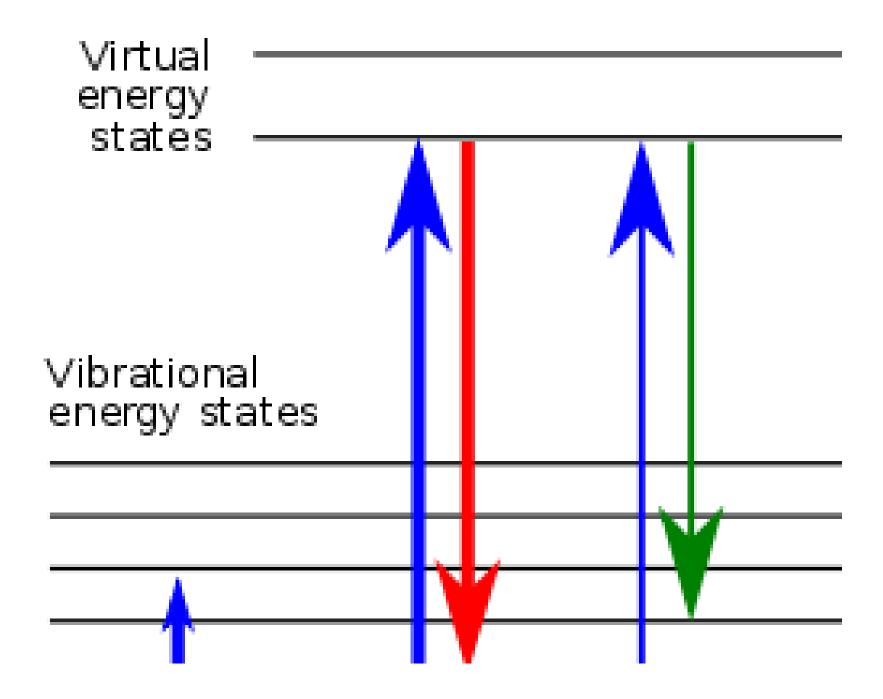
Elastic : Raleigh Scattering ; Inelastic: Change in frequency

(Stokes and Anti-stokes)









#### **Classical Theory of the Raman Effect: Molecular Polarizability**

- Spatial charge separation under influence of electri
  - $\rightarrow$  induced dipole moment  $\mu$ :

μ = α Ε

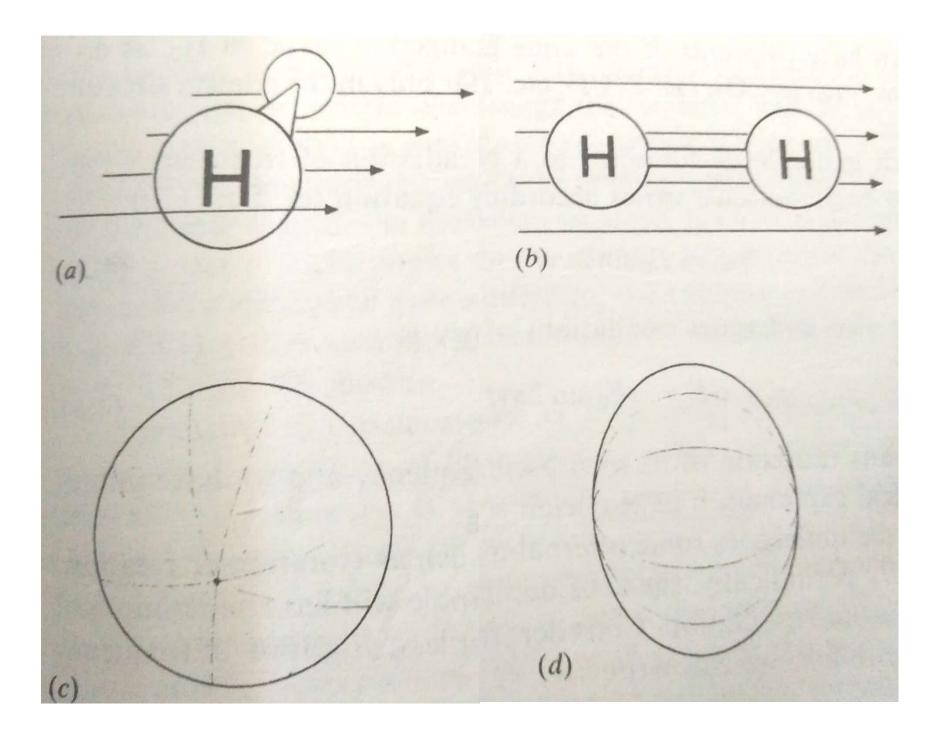
α: polarizability

Electric field *E* due to electromagn. wave with frequ

 $\boldsymbol{E} = \boldsymbol{E}_{\mathbf{0}} \cos 2\pi \, \boldsymbol{v}_0 t$ 

$$\rightarrow \mu = \alpha E_0 \cos 2\pi v_0 t$$

- Amission of light at same frequency V



- Internal vibrational motion with Eigenfrequency  $v_M$  $q = q_0 \cos 2\pi v_M t$
- Polarizability  $\alpha \rightarrow$  develop in series

 $\alpha = \alpha_{q=0} + (\partial \alpha / \partial q)_{q=0} q + higher order terms$ 

- $\rightarrow \mu = \alpha E$ 
  - $= (\alpha_{q=0} + (\partial \alpha / \partial q)_{q=0} q_0 \cos 2\pi v_M t) E_0 \cos 2\pi$  $= \alpha_{q=0} E_0 \cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} q_0 E_0 [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial q)_{q=0} [\cos 2\pi v_0 t + \frac{1}{2} (\partial \alpha / \partial$

## → harmonic oscillator: $\Delta v = \pm 1$

inelastic impact	elastic impact	in
inclastic impact	clastic impact	
ΔE <sub>M</sub> > 0	ΔE <sub>M</sub> = 0	
$v = v_0 - v_{vib} < v_0$	$v = v_0$	V
$hv_0$ $hv_0$ $h(v_0 - v_{vib})$	$\frac{hv_{0}}{hv_{0}}$	

