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On a comprehensive model of the novel coronavirus (COVID-19) under Mittag-Leffler derivative

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Highlights

- We investigate the Coronavirus (COVID-19) mathematical model spread in Wuhan Hubei Province of China recently by a novel fractional nonlocal operator (ABC).
- We study two aspects of the Model that: Qualitative aspect and Numerical aspect.
- Existence and Ulam-Hyers stability results of the model are first study by fixed point theory.
- Numerical aspect is then study by a novel method (fractional Adams Bashforth method).
- Graphical results are displayed to understand the method.

Abstract

The major purpose of the presented study is to analyze and find the solution for the model of nonlinear fractional <u>differential equations</u> (FDEs) describing the deadly and most parlous virus so-called coronavirus (COVID-19). The mathematical model depending of fourteen nonlinear FDEs is presented and the corresponding numerical results are studied by applying the fractional Adams Bashforth (AB) method. Moreover, a recently introduced fractional nonlocal operator known as Atangana-Baleanu (AB) is applied in order to realize more effectively. For the current results, the fixed point theorems of Krasnoselskii and Banach are hired to present the existence, uniqueness as well as stability of the model. For numerical simulations, the behavior of the approximate solution is presented in terms of graphs through various fractional orders. Finally, a brief discussion on conclusion about the simulation is given to describe how the transmission dynamics of infection take place in society.

Introduction

One of the greatest missions given to humankind is to control the environment within which they live. However, the growth of the human population speedily is increasing in relation to different cultures and ways of life. Although Humanity has evolved many species of devices and equipment to obtain a beautiful life. But this excessive development sometimes leads to disasters in the environment. Especially, the use of nutrients, carriages, mobiles, cosmetics, electrified and petroleum equipment, etc, making the environment highly contaminated and infused with viruses.

Newly, the whole world hardship a new coronavirus comprehensive and it was named (COVID-19) which was claimed to flowed first in Wuhan city, China. It has been considered that the origin of COVID-19 is the transportation from animal to human as numerous infected status claimed that they had been to a local fish and wild animal market in Wuhan on 28 November [1]. Soon, some investigators assured that transportation also occurs person to other [2]. This virus dates back to 1965 when Tyrrell and Bynoe were identified when they passage a virus so-called B814 [3]. This virus is found in the cultures of the organs of the human embryonic trachea acquired by the respiratory system of an adult [4].

Because infectious diseases are a major threat to humans as well as the country's economy. A proper understanding of the dynamics of the disease plays a significant role in reducing infection in society.

Application of an appropriate strategy against the disease transportation is another challenge. The mathematical modeling tactic is one of the main tools in order to handle these challenges. Many disease models have been evolved in the existing literature which authorizes us to explore and dominance the prevalence of infectious diseases in a better style. Most of these models rely on classical differential equations. However, in the past few years, it is paying attention that fractional differential equations can be applied to model global phenomena with a greater grade of precision and its applications can be found in various fields for instance dynamic, biology, engineering, control theory, economics, finance and in epidemiology.

The Fractional calculus transacts with differentiation and integration involving fractional order, which is further outstanding and beneficial than the ordinary integer order in the explanation of the real-world problems, also in the modeling of real phenomena due to a characterization of the memory and hereditary properties [5], [6], the integer-order derivative doesn't familiarize the dynamics among two various points. Various types of fractional order or nonlocal derivatives were proposed in the present literature to transact the reduction of a traditional derivative. For instance, based on power-law, Riemann-Liouville introduced the idea of fractional derivative. After that Caputo-Fabrizio in [7] have proposed a new fractional derivative utilizing the exponential kernel. This derivative has a few troubles related to the locality of the kernel. Newly, to overcome Caputo-Fabrizio's problem, Atangana and Baleanu (AB) in [8] have proposed a new modified version of a fractional derivative with the aid of popularized Mittag-Leffler function (MLF) as nonsingular kernel and nonlocal. Since the generalized MLF is used as the kernel and it's guaranteed no singularity, the AB fractional derivative supply a stellar description of memory [9], [10], [11].

Because of mathematical models, we can know the rate of change in the COVID-19 how the disease can affect people at risk and quarantined. The area dedicated to the study of the biological model of infectious diseases is a warm area for recent research. Numerous studies on mathematical models were presented to the study of stability theory and the results of existence and improvement of biological models, see [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24]. For example, Chen, et al., in [12] presented a big mathematical model for simulating the phase-based transmissibility of a novel coronavirus. Hussain et al., in [13] showed the existence and uniqueness of results for the fractional model involving fractional derivative in Atangana-Baleanu sense using Schaefer's and Banach's fixed point theorems. Moreover, they applied the Shehu transform and Picard method to explore the iterative solutions and its stability for the proposed fractional model. A conceptual model for the COVID-19 disease, which effectively catches the time line of this virus outbreak was proposed by Lin et. al. in [14]. In [16] A mathematical modeling and dynamics of a novel (2019-nCoV) are formulated under a fractional model with considering the available infection cases within 7 days. A mathematical model considered by Shaikh et al. [15] to study estimate of the effectiveness of prophylactic measures, prophesying future outbreaks and possibility control strategies of COVID-19.

Due to the success of this operator in modeling infectious diseases, and motivated by the above useful applications of some fractional operators in epidemic mathematical models, in this paper we are studying the dynamics of the novel coronavirus model suggested by Chen et.al., [12] in the form of the system of the nonlinear differential equations involving the AB Caputo fractional derivative:

$$\begin{pmatrix} ABC D_{0+}^{\infty} \mathscr{S}_{B}(t) = \Lambda_{B} - m_{B}\mathscr{S}_{B} - \beta_{B}\mathscr{S}_{B}\mathscr{S}_{B}, \\ ABC D_{0+}^{\infty} \mathscr{S}_{B}(t) = \beta_{B}\mathscr{S}_{B}\mathscr{S}_{B} - m_{B}\mathscr{S}_{B}, \\ ABC D_{0+}^{\infty} \mathscr{S}_{B}(t) = m_{B}\mathscr{S}_{B} - (\gamma_{B} + m_{B})\mathscr{I}_{B}, \\ ABC D_{0+}^{\infty} \mathscr{S}_{B}(t) = \gamma_{B}\mathscr{S}_{B} - m_{B}\mathscr{R}_{B}, \\ ABC D_{0+}^{\infty} \mathscr{S}_{H}(t) = \Lambda_{H} - m_{H}\mathscr{S}_{H} - \beta_{BH}\mathscr{S}_{H}\mathscr{S}_{B} - \beta_{H}\mathscr{S}_{H}\mathscr{S}_{H}, \\ ABC D_{0+}^{\infty} \mathscr{S}_{H}(t) = \beta_{BH}\mathscr{S}_{H}\mathscr{S}_{B} + \beta_{H}\mathscr{S}_{H}\mathscr{S}_{H} - m_{H}\mathscr{S}_{H}, \\ ABC D_{0+}^{\infty} \mathscr{S}_{H}(t) = w_{H}\mathscr{S}_{H} - (\gamma_{H} + m_{H})\mathscr{S}_{H}, \\ ABC D_{0+}^{\infty} \mathscr{S}_{H}(t) = \gamma_{H}\mathscr{S}_{H} - m_{H}\mathscr{R}_{H}, \qquad \text{with the initial conditions} \\ ABC D_{0+}^{\infty} \mathscr{S}_{H}(t) = \gamma_{H}\mathscr{S}_{H} - m_{H}\mathscr{R}_{H}, \qquad \text{with the initial conditions} \\ ABC D_{0+}^{\infty} \mathscr{S}_{H} = \Lambda_{\rho} - m_{\rho}\mathscr{S}_{\rho} - \beta_{\rho}\mathscr{S}_{\rho}(\mathscr{I}_{\rho} + k\mathscr{A}_{\rho}) - \beta_{w}\mathscr{S}_{\rho}\mathscr{W} \\ - (1 - \delta_{\rho}) w_{\rho}\mathscr{E}_{\rho} - \delta_{\rho}w'_{\rho}\mathscr{E}_{\rho} - m_{\rho}\mathscr{E}_{\rho}, \\ ABC D_{0+}^{\infty} \mathscr{S}_{\rho} = (1 - \delta_{\rho}) w_{\rho}\mathscr{E}_{\rho} - (\gamma_{\rho} + m_{\rho})\mathscr{I}_{\rho}, \\ ABC D_{0+}^{\infty} \mathscr{R}_{\rho} = \gamma_{\rho}\mathscr{I}_{\rho} + \gamma'_{\rho}\mathscr{I}_{\rho} - m_{\rho}\mathscr{R}_{\rho}, \\ ABC D_{0+}^{\infty} \mathscr{R}_{\rho} = \gamma_{\rho}\mathscr{I}_{\rho} + \gamma'_{\rho}\mathscr{I}_{\rho} - m_{\rho}\mathscr{R}_{\rho}, \\ ABC D_{0+}^{\infty} \mathscr{R}_{\rho} = \gamma_{\rho}\mathscr{I}_{\rho} + \gamma'_{\rho}\mathscr{I}_{\rho} - m_{\rho}\mathscr{R}_{\rho}, \\ ABC D_{0+}^{\infty} \mathscr{R}_{\rho} = 0, \mathscr{S}_{R} = 0, \\ \mathscr{S}_{H} = 0 = \mathscr{S}_{H_{0}} \geq 0, \quad \mathscr{S}_{H} = 0 = \mathscr{S}_{H_{0}} \geq 0, \quad \mathscr{I}_{H} = 0 = \mathscr{I}_{H_{0}} \geq 0, \\ \mathscr{I}_{H} = 0 = \mathscr{I}_{H_{0}} \geq 0, \quad \mathscr{I}_{H} = 0 = \mathscr{I}_{H_{0}} \geq 0, \qquad (Mere ABC D_{0+}^{\infty} denotes the advection of the following terms of the following term$$

Atangana-Baleanu-Caputo fractional derivative of order ∝. The model is based on the following facts:

- 1. The bats were split into four closets: \mathscr{S}_B is a susceptible class of bats, \mathscr{E}_B is an exposed class of bats, \mathscr{I}_B is an infected class of bats, and \mathscr{R}_B is a removed class of bats.
- 2. The hosts were split into four classes: \mathscr{S}_H is a susceptible class of hosts, \mathscr{E}_H is an exposed class of hosts, \mathscr{I}_H is an infected class of hosts, and \mathscr{R}_H is a removed class of hosts.
- The people were split into six classes: 𝒫_ρ is a susceptible class of people, 𝒫_ρ is an exposed class of people, 𝒫_ρ is an infected class of people, 𝒫_ρ is asymptomatic infected people, 𝒫_ρ is a population of removed people due to death, and 𝒜 is the population of the virus in reservoir various.

 $\mathscr{S}_{B_0}, \mathscr{E}_{B_0}, \mathscr{I}_{B_0}, \mathscr{R}_{B_0}, \mathscr{S}_{H_0}, \mathscr{E}_{H_0}, \mathscr{I}_{H_0}, \mathscr{R}_{\rho_0}, \mathscr{E}_{\rho_0}, \mathscr{I}_{\rho_0}, \mathscr{A}_{\rho_0}, \mathscr{R}_{\rho_0}, \text{and } \mathscr{W}_0$ are the initial values corresponding to the three categories.

The parameters of the model (1) are described as follows:

 Λ_B denote the birth rate of bats, m_B is the death rate of bats, and β_B is the transmission rate from \mathscr{I}_B to \mathscr{S}_B . The parameters $\frac{1}{w_B}$, $\frac{1}{\gamma_B}$, $\frac{1}{w'_{\rho}}$, and $\frac{1}{\gamma'_{\rho}}$ are the incubation period of bats, the infectious period of bats, the latent period of people, and the infectious period of asymptomatic infection of people, respectively. Λ_H is Recruitment from total number of hosts and m_H The death rate of hosts. The symbols β_H and β_{BH} represent the transmission rate from \mathscr{I}_H to \mathscr{I}_H and from \mathscr{I}_B to \mathscr{I}_H , respectively. Λ_{ρ} and m_{ρ} are the birth and death rate parameter of people, respectively. In the proposed model β_{ρ} and κ are define the transmission rate from \mathscr{I}_p to \mathscr{I}_p and multiple of the transmissibility of \mathscr{A}_p to \mathscr{I}_p , respectively. β_{ω} and β_{ρ} means the transmission rate from \mathscr{W} to \mathscr{I}_p and from \mathscr{I}_p to \mathscr{I}_p , respectively. δ_{ρ} is the proportion of asymptomatic infection rate of people, γ_{ρ} is the infectious period of symptomatic infection of people and *a* is the retail purchases rate of the hosts in the market. The shedding coefficients from \mathscr{I}_ρ to \mathscr{W} and the shedding coefficients from \mathscr{I}_ρ to \mathscr{W}_H are denoted by μ_{ρ} and μ'_{ρ} , respectively. Finally, $\frac{1}{\varepsilon}$ is the lifetime of the virus in \mathscr{W} and \mathscr{N}_H is the total number of hosts.

The major aim of the paper is to demonstrate the existence, uniqueness and Ulam stability of solution for the model (1)-(2) by using some fixed point techniques. Moreover, the numerical simulations via the fractional version of Adams Bashfully technique to approximate the ABC fractional operator are performed, which it is shown that the model displays rich dynamical behaviors through graphical representation of numerical solutions.

This paper is coordinated as follows: Section 1 transacts with the introduction which contains a survey of the literature. Section 2 consists of some foundation preliminaries related the fractional calculus and nonlinear analysis. The existence and Ulam stability results on a proposed model are obtained in Sections 3 and 4. The numerical solution and numerical simulations of the model at hand are presented in Section 5. For the numerical simulation, we use a powerful two step numerical tool called fractional AB method. The concerned numerical method is more powerful than usual Euler method as well Taylor method. Because the mentioned method is faster convergent and stable as compared to Taylor and Euler method which are slowly convergent. For detail about this method, see [32], [33], [34], [35], [36].

Section snippets

Preliminaries

For short, we will use the following notations

$$\begin{aligned} \boldsymbol{\mathcal{O}}_{B} &= (\mathcal{S}_{B}, \mathcal{E}_{B}, \mathcal{I}_{B}, \mathcal{R}_{B}), \ \boldsymbol{\mathcal{O}}_{H} &= (\mathcal{S}_{H}, \mathcal{I}_{H}, \mathcal{R}_{H}), \\ \boldsymbol{\mathcal{O}}_{\rho} &= (\mathcal{S}_{\rho}, \mathcal{E}_{\rho}, \mathcal{I}_{\rho}, \mathcal{A}_{\rho}, \mathcal{R}_{\rho}, \mathcal{W}). \\ \Psi &= (\mathcal{O}_{B}, \mathcal{O}_{H}, \mathcal{O}_{\rho}) = (\mathcal{S}_{B}, \mathcal{E}_{B}, \mathcal{I}_{B}, \mathcal{R}, \mathcal{S}_{H}, \mathcal{E}_{H}, \mathcal{I}_{H}, \mathcal{R}_{H}, \mathcal{S}_{\rho}, \mathcal{E}_{\rho}, \mathcal{I}_{\rho}, \\ \mathcal{A}_{\rho}, \mathcal{R}_{\rho}, \mathcal{W}) \end{aligned}$$
 For the next analysis,

we define Banach space for onward analysis. Let $0 \le t \le T < \infty$, we define the Banach space by using J = [0,T] as $F = C(J, R^{14})$ under the supremum norm given by $\|\Psi\| = \sup_{t \in J} \left\{ |\Psi(t)| : \Psi \in F \right\}$ where $|\mathcal{O}_B(t)| = |\mathcal{S}_B(t)| + |\mathcal{E}_B(t)| + |\mathcal{I}_B(t)| + |\mathcal{R}_B(t)|$, $|\mathcal{O}_H(t)| = |\mathcal{S}_H(t)| + |\mathcal{E}_H(t)| + |\mathcal{I}_H(t)| + |\mathcal{R}_H(t)|$, $|\mathcal{O}_\rho(t)| = |\mathcal{S}_\rho(t)| + |\mathcal{E}_\rho(t)| + |\mathcal{I}_\rho(t)| + |\mathcal{A}_\rho(t)| + |\mathcal{R}_\rho(t)| + |\mathcal{W}(t)|$, and $\mathcal{S}_{\theta},...$

Existence of solutions for the proposed model (1)-(2)

Now, we debate the existence and uniqueness results of the model (1)-(2) by utilizing the fixed-point technique. Let us reformulated model (1) in the subsequent appropriate form

$$\begin{split} ^{ABC} \mathbb{D}_{0^+}^{\propto} \mathscr{S}_B \left(t \right) &= \mathscr{F}_1 \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{E}_B \left(t \right) &= \mathscr{F}_2 \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{I}_B \left(t \right) &= \mathscr{F}_3 \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{R}_B \left(t \right) &= \mathscr{F}_4 \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{S}_H \left(t \right) &= \mathscr{F}_5 \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{I}_H \left(t \right) &= \mathscr{F}_7 \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{I}_H \left(t \right) &= \mathscr{F}_8 \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{I}_P \left(t \right) &= \mathscr{F}_9 \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{I}_\rho \left(t \right) &= \mathscr{F}_{10} \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{I}_\rho \left(t \right) &= \mathscr{F}_{11} \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ ^{ABC} \mathbb{D}_{0^+}^{\infty} \mathscr{I}_\rho \left(t \right) &= \mathscr{F}_{11} \left(t, \mathscr{O}_B, \mathscr{O}_H, \mathscr{O}_\rho \right), \\ \end{array}$$

Ulam-Hyers stability

The notion of Ulam stability was initiated by Ulam [26], [27]. Then aforesaid stability has been scrutinized for classical fractional derivatives in many of the research articles, we refer to some of them like [28], [29], [30], [31]. Additionally, since stability is a prerequisite in respect of approximate solution, so we endeavor on Ulam type stability for the model (1) via using nonlinear functional analysis.

Definition 4.1

System (1)-(2) is U-H stable if there exists $\lambda > 0$ with the following property: For...

•••

Numerical approach

 $(\mathcal{Q}_{-}(+))$

In this part, we give approximation solutions of the ABC fractional model (1)-(2). Then the numerical simulations are acquired via the suggested scheme. To this aim, we employ the modified fractional version for Adams Bashforth technique to approximate the fractional integral in the sense AB.

Using the initial conditions and fractional integral operator, we convert model (1) into the integral

equation

Conclusion

This manuscript has been devoted to comprehensively investigate a mathematical model for calculating the transmissibility of novel Coronavirus (COVID-19) disease by using nonsingular fractional order derivative. The existence and uniqueness of the considered model has been guaranteed by applying Krasnoselskii and Banach fixed point theorems. Also some stability results of Ulam type have been constructed. With the help of fractional Adams Bashforth method method we have simulated the results...

CRediT authorship contribution statement

Mohammed S. Abdo: Writing - original draft. Kamal Shah: Methodology. Hanan A. Wahash: Formal analysis. Satish K. Panchal: Writing - review & editing....

Declaration of Competing Interest

We declare that none of the author has the competing or conflict of interest....

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