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### Article

The Minkowski's inequalities via  $\psi$ -Riemann–Liouville fractional integral operators

July 2020 · Rendiconti del Circolo Matematico di Palermo 70(2)

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### Abstract

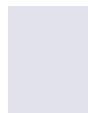
The main objective of this paper is to establish some new fractional integral inequalities of Minkowski's type by using  $\psi$ -Riemann–Liouville fractional integral operator, which is the classical Riemann–Liouville fractional integral of any function with respect to another function. Further, we establish some new fractional inequalities related to the reverse Minkowski's type inequality via  $\psi$ -Riemann–Liouville fractional integral operator. Using this fractional integral operator, some more integral inequalities of reverse Minkowski's type are also obtained.

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... for one parameter. The two-parameter fractional integral version of Inequality (4) for functional bounds was given by Aljaaidi and Pachpatte (2020) in [36] by using the Katugampola fractional integral; in 2020, they also presented the same inequality for functional bounds by using the  $\psi$ -Riemann-Liouville fractional integral (see [37]). Another inequality that is beneficial for this article is the Pólya-Szegő inequality, which was introduced by Pólya and Szegő [38] in 1925 as follows: ...

... for one parameter. The two-parameter fractional integral version of Inequality (4) for functional bounds was given by Aljaaidi and Pachpatte (2020) in [36] by using the Katugampola fractional integral; in 2020, they also presented the same inequality for functional bounds by using the  $\psi$ -Riemann-Liouville fractional integral (see [37]). Another inequality that is beneficial for this article is the Pólya-Szegő inequality, which was introduced by Pólya and Szegő [38] in 1925 as follows: ...

**(k,  $\psi$ )-Proportional Fractional Integral Pólya-Szegő and Grüss-Type Inequalities**

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Oct 2021

Tariq Aljaaidi · Deepak Pachpatte · Mohammed S. Abdo · Saleh S. Redhwan

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... . Minkowski's inequality is one of the inequalities that has been given the most attention in recent years. To cite just a few examples, directly linked to fractional operators, we recommend consulting [1, [8] [9][10][11][12][13][14][15] and references cited therein. ...

... It is clear, then, that the results obtained in [?, ?, 20] achieved within the framework of the Riemann integral [10], where we worked with the generalized k-fractional integrals [15], in the framework of integral Katugampola [8], with the  $\psi$ -integral among others, can be obtained as particular cases of our results. ...

**The Minkowski Inequality for Generalized Fractional Integrals**

Article Full-text available

Jan 2021

Juan Gabriel Galeano Delgado · Juan Eduardo Nápoles Valdes · Edgardo Pérez Reyes · Miguel Vivas-Cortez

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... where  $\phi$  and  $\psi$  are two integrable functions on  $[a, b]$ . Many researchers have given considerable attention to the both functionals and a number of inequalities and a number of extensions, generalizations and variants have appeared in the literature, for more details see ([1], [2] [11], [12], [17], [18], [19]). If  $\phi$  and  $\psi$  satisfies the following

condition  $(\phi(\tau) - \phi(\gamma))(\psi(\tau) - \psi(\gamma)) \geq 0$ , for any  $\tau, \gamma \in [a, b]$ , then  $\phi$  and  $\psi$  are synchronous on  $[a, b]$ , moreover,  $T(\phi, \psi, g, h) \geq 0$  (see also [16]). ...

... In this section, we consider the extended Chebyshev functional in case of synchronous functions (2). To prove our theorem in this section, we need the following lemma:

...

#### ON GENERALIZATION OF SOME INEQUALITIES OF CHEBYSHEV'S FUNCTIONAL USING GENERALIZED...

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... These inequalities involve fractional integrals and provide bounds on the functions being integrated. For example, several integral inequalities, such as the Grüss inequality, 28,29 Hölder's inequality [30][31][32] and Minkowski's inequality, 33, 34 can be extended to include fractional integrals. Inequalities involving both fractional integrals and derivatives are also studied in fractional calculus. ...

#### ON A NEW $\alpha$ -CONVEXITY WITH RESPECT TO A PARAMETER: APPLICATIONS ON THE MEANS AND...

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... There are many inequalities that are derived using convexity; for example, see the books [1,2]. See the works [3][4][5][6][7][8][9][10][11][12][13][14] for further information on the applications of inequality to diverse areas of mathematics, such as numerical analysis, probability density functions, and optimization. It should be noted that L'Hospital and Leibniz first proposed the concept of fractional calculus in 1695. ...

#### New Versions of Fuzzy-Valued Integral Inclusion Over p-Convex Fuzzy Number-Valued Mappings and Related Fuz...

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... Many inequalities derived using convexity exist in the literature; for example, see the books [25,31]. Inequalities have various applications to different branches of mathematics, including numerical analysis, probability density functions, and optimization; see the papers [2, 6,8,13,26,27,34]. ...

#### Hermite-Hadamard-type fractional-integral inequalities for (p, h)-convex fuzzy-interval-valued mappings

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... Inequalities have applications in many fields, such as analysis, optimization and the probability theory. For further information, we refer the reader to the papers [8, 9,12,17,25,26,35]. The inequality that has attracted

the most attention in the math community is the Hermite-Hadamard inequality [16]. ...

#### Hermite Hadamard Type Inequalities Involving $(k-p)$ Fractional Operator with $(\alpha, h - m) - p$ convexity

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 Vuk Stojiljković





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... After that, Rassias [14] in 1978 expanded the UH stability to create a new type of stability known as Ulam-Hyers-Rassias (UHR) stability. We point the reader to a number of papers for some recent stability analysis results [15,16, 17, 9,11,18,19,20]. For results on the existence and uniqueness of several kinds of initial value problems for FDEs involving the  $\psi$ -Hilfer derivative operator, one can see [21,15,22,23,24]. ...

#### On the Explicit Solution of $\psi$ -Hilfer Integro-Differential Nonlocal Cauchy Problem

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 Khaled A. Aldwoah ·  Mohammed A. Almalahi ·  Satish K Panchal ·  Mansour Saleman M. Lotayif


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... which completes the proof. (20), we obtain the reverse Minkowski's inequality for  $\psi$ -Riemann-Liouville fractional integral [5]: ...

#### Hölder and Minkowski type inequalities for pseudo-fractional integral

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... A variety of applications of convex inequalities exist in, for example, the fields of numerical analysis, physics and optimization problems. The following books can be referred to for more information [4][5][6][7] [8] [9][10][11] [12]. ...

#### Some Novel Inequalities for LR- $(k,h-m)$ - $p$ Convex Interval Valued Functions by Means of Pseudo Order Relation

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 Vuk Stojiljković ·  Rajagopalan Ramaswamy ·  Ola. A. Ashour ·  Stojan N Radenović





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... Various inequalities were derived as consequences of the famous Jensen's inequality; see [1,2]. Convex inequalities have many applications, for example, in probability theory, analysis, and optimization problems [3] [4][5][6] [7] [8][9]. See the following books for further information [10,11]. ...

#### Riemann-Liouville Fractional Inclusions for Convex Functions Using Interval Valued Setting

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 Vuk Stojiljković ·  Rajagopalan Ramaswamy ·  Ola. A. Ashour ·  Stojan N Radenović

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... Inequalities have various applications to analysis problems, optimization, probability theory, etc. For applications, we refer readers to the papers [3][4][5][6] [7] [8][9]. One of the most elegant results in the theory of convex inequalities is the Hermite-Hadamard inequality [10]. ...

#### Hermite–Hadamard Type Inequalities Involving (k-p) Fractional Operator for Various Types of Convex Functions

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... In view of the foregoing, it is natural for mathematicians to develop and explore fractional operators that are generalized categories of the current specific cases. For some applications and recent contributions of fractional calculus, we refer the readers to [7][8][9][10] [11]. ...

#### Generalized proportional fractional integral Hermite–Hadamard's inequalities

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... In 2019, Rahman et al. [25] presented the reverse Minkowski's inequality, and they also established some other fractional integral inequalities by using generalized proportional fractional integral operators. In 2020, the reverse Minkowski inequalities were studied by Aljaaidi and Pachpatte [26] via the Riemann-Liouville operator with respect to the positive monotone function  $\phi$ . In the same year, Rashid et al. [27] presented a note on reverse Minkowski inequalities by using a generalized proportional fractional operator involving another function. ...

#### Generalized proportional fractional integral functional bounds in Minkowski's inequalities

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... Inequalities have a fascinating numerical model due to their important applications in classical as well as fractional calculus and mathematical analysis. For applications, we refer readers to the papers [1][2][3][4] [5] [6][7]. In such a scenario, the Hermite-Hadamard inequality [8] is undoubtedly one of the most elegant results. ...

#### Hermite-Hadamard Type Inequalities Involving k-Fractional Operator for (h, m)-Convex Functions

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... Thereafter, the Ulam-Hyers stability was extended by Rassias [19] in 1978 to a new type of stability which called Ulam-Hyers-Rassias stability. For some recent results of stability analysis, we refer the reader to a series of papers [2, 13, 16, 17, 23, 25, 30, 31]. For the existence and uniqueness results of different classes of initial value problem for fractional differential equations involving  $\psi$ -Hilfer derivative operator, one can see [1, 2, 3, 14, 26]. ...

#### On the Explicit Solution of $\psi$ -Hilfer Integro-Differential Nonlocal Cauchy Problem

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... There is a prominent and noticeable interest in the investigation of qualitative characteristics of solutions (existence, uniqueness, stability) of FDEs. For applications and recent work, we refer the readers to [4, 7, 14, 18, 37, 42, 43]. ...

#### Some properties of implicit impulsive coupled system via $\phi$ -Hilfer fractional operator

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Jul 2021 · BOUND VALUE PROBL

 Mohammed A. Almalahi ·  Satish K Panchal

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... The mathematical inequalities play a very reliable role in classical integral and differential equations as well as in the past few years, many of useful mathematical inequalities have been originated by many authors, see [5] [6] [7][8]. One of the most significant integral inequalities is that discovered by Hermite [9] and Hadamard [10] for convex function  $f$  as follows ...

#### Reverse Hermite-Hadamard's inequalities using $\psi$ -fractional integral

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Dec 2020 · EML

 Tariq Aljaaidi ·  Deepak Pachpatte

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... Rashid et al. [32,33] investigated new fractional integral operator known Generalized proportional fractional integral operators with respect to another function and established certain Grüss type and the reverse Minkowski type inequalities for this operators. Aljaaidi with Pachpatte [3], presented Minkowski inequalities by mean of  $\psi$ -Riemann Liouville fractional integral operators. A large variety of inequalities have been proposed and studied for different fractional operators, see [11, 31, 35, 36] and the references therein. ...

#### $\psi$ -Caputo Fractional Iyengar's Type Inequalities

Preprint

Sep 2020





 Bhagwat Yewale ·  Deepak Pachpatte

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#### Some Hermite-Hadamard-Type Fractional Integral Inequalities Involving Twice-Differentiable Mappings

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**New generalization of reverse Minkowski's inequality for fractional integral**

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Article

## Weighted Norm Inequalities for Certain Integral Operators. II

March 1985 · Proceedings of the American Mathematical Society

Hans P. Heinig

Conditions on nonnegative weight functions  $u(x)$  and  $v(x)$  are given which ensure that an inequality of the form  $(\int |Tf(x)|^q u(x) dx)^{1/q} \leq C (\int |f(x)|^p v(x) dx)^{1/p}$  holds for  $1 \leq q < p < \infty$ , where  $T$  is an integral operator of the form  $\int_{-\infty}^{\infty} K(x,y) f(y) dy$  or  $\int_{-\infty}^{\infty} K(y,x) f(y) dy$  and  $C$  a constant independent of  $f$ . Specifically a number of inequalities for well-known classical operators are obtained. Inequalities ... [\[Show full abstract\]](#)

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## Some New Fractional Inequalities for Coordinated Convexity over Convex Set Pertaining to Fuzzy-Numbe...

October 2023

● Muhammad Bilal Khan · Tareq Saeed · Adriana Cătaș · Ahmed Alshehri

In this study, we first discover some new concept coordinated UD-convex mappings with fuzzy-number values. After that, we look into Hermite-Hadamard type inequalities via fuzzy-number-valued coordinated UD-convex fuzzy-number-valued mapping (coordinated UD-convex FNVM). In the case of coordinated UD-convex FNVM, novel conclusions are derived by making particular decisions in recently proven ... [\[Show full abstract\]](#)

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## A note on the main theorem for absolutely monotonic functions

February 2012

● Sergei Sitnik

We prove by a simple counterexample that the main theorem for absolutely monotonic functions on  $(0, \infty)$  from the book Mitrinović D.S., Pečarić J.E., Fink A.M. "Classical And New Inequalities In Analysis", Kluwer Academic Publishers, 1993, is not valid.

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Article

[Full-text available](#)Generalizations of Hermite-Hadamard-Fejer Type Inequalities for Functions Whose Derivatives are  $s$ -Co...

October 2014 · Turkish Journal of Analysis and Number Theory

● Erhan Set · ● İmdat İşcan · ● İker Mumcu

In this work, the new results related to right hand side of Hermite-Hadamard-Fejer inequality for  $s$ -convex functions in the second sense via fractional integrals are obtained. This results are generalization of the results obtained by İşcan in [17].

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