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Coefficient Estimates for a Subclass of Bi-univalent Functions Defined by Sălăgean Type q -Calculus Operator

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Abstract

In this paper, we introduce and investigate a new subclass of bi-univalent functions defined by Sălăgean q -calculus operator in the open disk \mathbb{U} . For functions belonging to the subclass, we obtain estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. Some consequences of the main results are also observed.
 Keywords: analytic functions, bi-univalent functions, coefficient bounds, Sălăgean q -differential operator, Sălăgean derivative.

1. Introduction

Let \mathcal{A} denote the family of functions analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and $|z| < 1$, which are normalized by the condition: $f(0) = f'(0) = 1 = 0$ and given by the following Taylor-Maclaurin series (1.1) $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$.

Also let \mathcal{S} be the class of functions $f \in \mathcal{A}$ of the form given by (1.1), which are univalent in

\mathbb{U} . The Koebe one-quarter theorem [7] ensures that the image of \mathbb{U} under every univalent function

$f \in \mathcal{S}$ contains a disk of radius $\frac{1}{4}$. Hence every function $f \in \mathcal{S}$ has an inverse

f^{-1} , defined by $f^{-1}(f(z)) = z$, ($z \in \mathbb{U}$) and $f^{-1}(f(w)) = w$, ($|w| < r(f) = r(f) \geq \frac{1}{4}$), where $f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3) w^3 - (5a_2 a_3 + a_4) w^4 + \dots$

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interesting examples of functions in the class Σ see [28] (see also [4]). From the work of Srivastava *et al.* [28], we choose to recall the following examples of functions in the class Σ ...

The class of bi-univalent functions was investigated by Lewin [3], who proved that $|a_2| < 1/2$. In 1981, Styer and Wright [30] showed that $|a_3| > 4/3$. Subsequently, Brannan and Clunie [3] improved Lewin's result to $|a_2| \leq 2$. Netanyahu [14], showed that $\max_{f \in \Sigma} |a_2| = 4/3$. In 1985, Branges [2] proved Bieberbach conjecture which showed that: $|a_n| \leq n$ ($n \in \mathbb{N}$), $n \geq 2$.

The problem of finding coefficient estimates for the bi-univalent functions has received much attention in recent years. In fact, the aforementioned work of Srivastava *et al.* [28] essentially revived the investigation of various subclasses of bi-univalent function class Σ in recent years and that it leads to a flood of papers on the subject (see, for e.g., [6, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29]). It was followed by such works as those by Tang *et al.* [31], Xu *et al.* [32, 33] and Lashin [12], and others (see, for e.g., [1, 5, 8]). The coefficient estimate problem involving the bound of $|a_n|$ ($n \in \mathbb{N}$) is still an open problem.

In the field of geometric function theory, various subclasses of the normalized analytic function on class A have been studied from different view points. The q -calculus as well as the fractional calculus provide important tools that have been used in order to investigate various subclasses of A .

Historically speaking, the firm footing of the usage of the q -calculus in the context of geometric function theory which was actually provided and q -hypergeometric functions were first used in geometric function theory in a book chapter by Srivastava (see, for details, [18, pp. 347 et seq.]). Ismail *et al.* [10] introduced the class of generalized complex functions via q -calculus on some subclasses of analytic functions. Recently, Purohit and Raina [16] investigated applications of fractional q -calculus operator to define new classes of functions which are analytic in unit disk U (see, for details, [9]).

For $0 < q < 1$, the q -derivative D_q of a function f given by (1.1) is defined as $(D_q f)(z) = (f(qz) - f(z))/(qz - z)$ for $z \neq 0$, $f(0)$ for $z = 0$.

We note that $\lim_{q \rightarrow 1^-} D_q f(z) = f'(z)$. From (2.2), we deduce that $(D_q^n f)(z) = \sum_{k=0}^{n-1} [k]_q! q^{k(k-1)/2} f^{(k)}(z)$, where as $q \rightarrow 1^-$, $[k]_q! \rightarrow k!$.

Making use of the q -differential operator for function $f \in A$, we introduced the Sălăgean q -differential operator as given below

$$(D_{q,\lambda} f)(z) = f(z) D_{q,\lambda} f(z) = z D_q (D_{q,\lambda} f)(z) = z D_q (D_{q,\lambda} f)(z) = z \sum_{k=0}^{\infty} [k]_q! q^{k(k-1)/2} a_{k+1} z^k \quad (n \in \mathbb{N}_0, z \in U).$$

We note that $\lim_{q \rightarrow 1^-} D_{q,\lambda} f(z) = D_{\lambda} f(z)$, $(n \in \mathbb{N}_0, z \in U)$, the familiar Sălăgean derivative [17].

Recently, Kamble and Shrikan [11] introduce the following two subclasses of the bi-univalent function class Σ and obtained estimate on first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in these subclasses as follows.

Definition 11 ([11])

For $0 < \alpha \leq 1$, $0 < q < 1$, $\lambda \geq 1$, $\mu \geq 0$ and $n \in \mathbb{N}_0$, a function $f(z)$ given by (1.1) is said to be in the class $\mathcal{H}_{\Sigma, q}(\mu, \alpha, \lambda)$ if the following conditions are satisfied

$$(1.7) f \in \Sigma \text{ and } |\arg\{(1-\lambda)(D_{q,\lambda} f(z))^{\mu} + \lambda(D_{q,\lambda} f(z))^{1-\mu}\}| < \alpha \pi/2$$

$$(1.8) |\arg\{(1-\lambda)(D_{q,\lambda} g(w))^{\mu} + \lambda(D_{q,\lambda} g(w))^{1-\mu}\}| < \alpha \pi/2,$$

where the function g is given by $(1.9) g(w) = w - a_2 w^2 + (2a_2^2 - a_3) w^3 - (5a_2^3 - 5a_2 a_3 + a_4) w^4 + \dots$ and $D_{q,\lambda}$ is the Sălăgean q -differential operator.

Theorem 12 ([11])

Let $f(z)$ given by (1.1) be in the function class $\mathcal{H}_{\Sigma, q}(\mu, \alpha, \lambda)$. Then

$$(1.10) |a_2| \leq \alpha [2]_q! \mu^{1/2} [3]_q! n^{-1/2} + \lambda^{1/2} [2]_q! [2]_q! [2]_q! n^{-1/2}$$

$$\text{and } |a_3| \leq \alpha [2]_q! \lambda^{1/2} [2]_q! [2]_q! n^{-1/2} + \alpha [2]_q! [3]_q! n^{-1/2}$$

where $0 < \alpha \leq 1$, $0 < q < 1$, $\lambda \geq 1$, $\mu \geq 0$ and $n \in \mathbb{N}_0$.

Definition 13 ([11])

For $0 \leq \beta < 1$, $0 < q < 1$, $\lambda \geq 1$, $\mu \geq 0$ and $n \in \mathbb{N}_0$, a function $f(z)$ given by (1.1) is said to be in the class $\mathcal{H}_{\Sigma, q}(\mu, \beta, \lambda)$ if the following conditions are satisfied

$$(1.12) f \in \Sigma \text{ and } \operatorname{Re}\{(1-\lambda)(D_{q,\lambda} f(z))^{\mu} + \lambda(D_{q,\lambda} f(z))^{1-\mu}\} > \beta$$

$$(1.13) \operatorname{Re}\{(1-\lambda)(D_{q,\lambda} g(w))^{\mu} + \lambda(D_{q,\lambda} g(w))^{1-\mu}\} > \beta.$$

Theorem 14 ([11])

Let $f(z)$ given by (1.1) be in the function class $\mathcal{H}_{\Sigma, q}(\mu, \beta, \lambda)$. Then

$$(1.14) |a_2| \leq \min\{\lambda(1-\beta), 2|\lambda+1-\mu|\} |z_1|^{2n+1} |z_2|^{2n+1} (2\lambda-\mu)^2 \beta^{1-\mu} |\mu|^{2n+1}$$

and





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By appropriately specializing the parameters in Definition 1.1 and 1.3, we can get several known subclasses of the bi-univalent function class Σ . For example:

- For $n = 0$ and $q \rightarrow 1^-$, we obtain the bi-univalent function classes $\mathcal{H}\Sigma_{1,1}(0, \alpha, \lambda; -\beta, \mu, \alpha, \lambda)$ and $\mathcal{H}\Sigma_{1,1}(0, \beta, \lambda; -\beta, \mu, \beta, \lambda)$ (see [21]).
- For $\mu = 1, n = 0$ and $q \rightarrow 1^-$, we obtain the bi-univalent function classes $\mathcal{H}\Sigma_{1,1}(n, \alpha, \lambda; -\beta, \mu, \alpha, \lambda)$ and $\mathcal{H}\Sigma_{1,1}(n, \beta, \lambda; -\beta, \mu, \beta, \lambda)$ (see [15]).
- For $\mu = 1, n = 0, \lambda = 1$ and $q \rightarrow 1^-$, we obtain the bi-univalent function classes $\mathcal{H}\Sigma_{1,1}(0, \alpha, 1; -\beta, \mu, \alpha, 1)$ and $\mathcal{H}\Sigma_{1,1}(0, \beta, 1; -\beta, \mu, \beta, 1)$ (see [28]).
- For $\mu = 0, n = 0, \lambda = 1$ and $q \rightarrow 1^-$, we obtain the bi-univalent function classes $\mathcal{H}\Sigma_{1,1}(0, \alpha, 1; -\beta, \mu, \alpha, 1)$ and $\mathcal{H}\Sigma_{1,1}(0, \beta, 1; -\beta, \mu, \beta, 1)$ (see [4]).

This paper is a sequel to some of the aforesaid works (especially see [11, 32, 33]). Here we introduce and investigate the general

subclass $\mathcal{H}\Sigma_{h,p}(\lambda, \mu, n, q)$ ($0 < q < 1, \lambda \geq 1, \mu \geq 0$) of the analytic function class \mathcal{A} , which is given by Definition 1.6 below.

Definition 1.6

Let $h: U \rightarrow \mathbb{C}$ be analytic functions and $\min\{\operatorname{Re}\{h(z)\}, \operatorname{Re}\{h(p)\}\} > 0$ ($z \in U$) and $h(0) = p$ ($0 < |p| < 1$).

Also let the function f given by (1.1) be in the analytic function class \mathcal{A} . We say that $f \in \mathcal{H}\Sigma_{h,p}(\lambda, \mu, n, q)$ ($0 < q < 1, \lambda \geq 1, \mu \geq 0$ and

- $n \in \mathbb{N}_0$) if the following conditions satisfied:
- (1.16) $f \in \Sigma$ and $(1-\lambda) \{D_q n f(z) \mu + \lambda \{D_q n f(z)\} \{D_q n f(z)\}^{\mu-1} \} \in h(U)$ ($z \in U$),
- and
- (1.17) $(1-\lambda) \{D_q n g(w) \mu + \lambda \{D_q n g(w)\} \{D_q n g(w)\}^{\mu-1} \} \in p(U)$ ($w \in U$),
- where the function g is given by (1.9).
- If $f \in \mathcal{H}\Sigma_{h,p}(\lambda, \mu, n, q)$, then
- (1.18) $f \in \Sigma$ and $|\arg\{(1-\lambda) \{D_q n f(z) \mu + \lambda \{D_q n f(z)\} \{D_q n f(z)\}^{\mu-1} \} - \arg p| < \alpha n \tau_2$
- and
- (1.19) $|\arg\{(1-\lambda) \{D_q n g(w) \mu + \lambda \{D_q n g(w)\} \{D_q n g(w)\}^{\mu-1} \} - \arg p| < \alpha n \tau_2$
- or
- (1.20) $f \in \Sigma$ and $\operatorname{Re}\{(1-\lambda) \{D_q n f(z) \mu + \lambda \{D_q n f(z)\} \{D_q n f(z)\}^{\mu-1} \} > \beta$
- and
- (1.21) $\operatorname{Re}\{(1-\lambda) \{D_q n g(w) \mu + \lambda \{D_q n g(w)\} \{D_q n g(w)\}^{\mu-1} \} > \beta$,

where the function g is given by (1.9). Our paper is motivated and stimulated especially by the work of Srivastava *et al.* [21, 28]. Here we propose to investigate the bi-univalent function subclass $\mathcal{H}\Sigma_{h,p}(\lambda, \mu, n, q)$ of the function class Σ and find estimates on the initial coefficients $|a_n|$ and $|a_{-n}|$ for functions n in the new subclass of the function class Σ using Sălăgean q -differential operator.

2. A Set of General Coefficient Estimates

In this section, we derive estimates on the initial coefficients $|a_n|$ and $|a_{-n}|$ for functions in subclass $\mathcal{H}\Sigma_{h,p}(\lambda, \mu, n, q)$ given by Definition 1.6.

Theorem 2.1

Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{H}\Sigma_{h,p}(\lambda, \mu, n, q)$. Then

$$(2.1) |a_2| \leq \min\{h^*(0) \{1 + \lambda \mu\} \{2|\lambda + 1 - \mu|\} |z_1|^{2n+1} |z_2|^{2n+1} (2\lambda - \mu)^2 \beta^{1-\mu} |\mu|^{2n+1} \}^{1/2}$$

$$(2.2) |a_2| \leq \min\{h^*(0) \{1 + \lambda \mu\} \{2|\lambda + 1 - \mu|\} |z_1|^{2n+1} |z_2|^{2n+1} (2\lambda - \mu)^2 \beta^{1-\mu} |\mu|^{2n+1} \}^{1/2}$$

Proof
It follows from (1.16) and (1.17) that
(2.3) $(1-\lambda) \{D_q n f(z) \mu + \lambda \{D_q n f(z)\} \{D_q n f(z)\}^{\mu-1} \} \in h(U)$
and
(2.4) $(1-\lambda) \{D_q n g(w) \mu + \lambda \{D_q n g(w)\} \{D_q n g(w)\}^{\mu-1} \} \in p(U)$
Comparing the coefficients of z and z^2 in (2.3) and (2.4), we have
(2.5) $(\lambda + \mu) |z_1|^{2n+2} - h_1 + (2\lambda + 1 - \mu) |z_1|^{2n+2} (2\lambda - \mu)^2 \beta^{1-\mu} |\mu|^{2n+1} |z_2|^{2n+2} - p_1$
and
(2.6) $(\lambda + \mu) |z_1|^{2n+2} (3\lambda + 4) |z_2|^{2n+2} - (2\lambda + 1 - \mu) |z_1|^{2n+2} (2\lambda - \mu)^2 \beta^{1-\mu} |\mu|^{2n+1} |z_2|^{2n+2} - p_2$
From (2.5) and (2.7), we obtain
(2.9) $h_1 - p_1$
and
(2.10) $2\lambda + \mu |z_1|^{2n+2} |z_2|^{2n+2} - h_2 + p_2$

Also, from (2.6) and (2.8), we find that
(2.11) $(\lambda + \mu) |z_1|^{2n+2} |z_2|^{2n+2} - (2\lambda + 1 - \mu) |z_1|^{2n+2} (2\lambda - \mu)^2 \beta^{1-\mu} |\mu|^{2n+1} |z_2|^{2n+2} - p_2$
Therefore, we find from the equations (2.10) and (2.11) that
 $|a_2| \leq h^*(0) \{1 + \lambda \mu\} \{2|\lambda + 1 - \mu|\} |z_1|^{2n+1} |z_2|^{2n+1}$
and
 $|a_2| \leq h^*(0) \{1 + \lambda \mu\} \{2|\lambda + 1 - \mu|\} |z_1|^{2n+1} |z_2|^{2n+1}$
respectively. So we get the desired estimate on the coefficients $|a_n|$ as asserted in (2.1).
Next, in order to find the bound on the coefficient $|a_{-2}|$, we subtract (2.8) from (2.6), we get

(2.12) $2\lambda \mu \{3\} q_n a_3 - 2\{3\} q_n \{2\lambda \mu a_2^2 - h_2 - p_2\}$.

Upon substituting the value of a_2 from (2.10) into (2.12), we arrive at

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(2.13) $\lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$.

(2.14) $\lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$.

Consequently, we have

(2.14) $\lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$.

This evidently completes the proof of Theorem 2.1.

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3. Corollaries and Consequences Go to

By Setting $\mu = 1$, $q \rightarrow 1^-$ and $n = 0$ in Theorem 2.1, we deduce the following consequence of Theorem 2.1.

Corollary 3.1

Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{B}\Sigma_h^*(\lambda)$ ($\lambda \geq 1$). Then

$$(3.1) \lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$$

and

$$(3.2) \lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$$

By Setting $\mu = 0$, $\lambda = 1$, $q \rightarrow 1^-$ and $n = 0$ in Theorem 2.1, we deduce the following.

Corollary 3.2 ([5])

Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{B}\Sigma_h^*$. Then

$$(3.3) \lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$$

and

$$(3.4) \lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$$

Remark 3.3

Corollary 3.2 is an improvement of the following estimates obtained by Xu *et al.* [33].

Corollary 3.4 ([33])

Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{B}\Sigma_h^*(\lambda)$ ($\lambda \geq 1$). Then

$$(3.5) \lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$$

and

$$(3.6) \lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$$

By Setting $\lambda = 1$, $\mu = 1$, $q \rightarrow 1^-$ and $n = 0$ in Theorem 2.1, we deduce the following Corollary 3.5.

Corollary 3.5 ([32])

Let the function $f(z)$ given by Taylor-Maclaurin series expansion (1.1) be in the function class $\mathcal{H}\Sigma_h^*$. Then

$$(3.7) \lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$$

and

$$(3.8) \lambda a_3 \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \mu \{2\} \{2\} q_n \{1\} h^{(0)} \{1\} p^{(0)} \{1\} q_n \{1\} \{3\} q_n$$

4. Concluding Remarks and Observations Go to

The main objective in this paper has been to derive first two Taylor-Maclaurin coefficient estimates for functions belonging to a new subclass $\mathcal{H}\Sigma_h^*(\lambda, \mu, n, q)$ of analytic and bi-univalent function in the open unit disk \mathbb{U} . Indeed, by using Sălăgean q -calculus

operator, we have successfully determined the first two Taylor-Maclaurin coefficient estimates for functions belonging to a new subclass $\mathcal{H}\Sigma_h^*(\lambda, \mu, n, q)$.

By means of corollaries and consequences which we discuss in the preceding section by suitable specializing the parameters λ and μ , we have also shown already that the results presented in this paper would generalize and improve some recent works of Xu, *et al.* [32, 33] and other authors.

Acknowledgements Go to

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