Non-linear state feedback control for uncertain systems using a finite time disturbance observer

Published: 26 May 2021

Volume 10, pages 427–434, (2022) Cite this article

International Journal of Dynamics and Control

Ajay Borkar & P. M. Patil

Abstract

This paper develops a non-linear state feedback control for a non-linear system affected by parametric uncertainty and external disturbance. The parametric uncertainty and external disturbance are estimated as a lumped disturbance using a finite time disturbance observer. By designing the non-linear state feedback control based on a finite time disturbance observer, the proposed method counters the effect of lumped disturbance and ensures the system states' finite-time convergence. The performance of the proposed scheme is compared with a sliding mode controller using a third-order non-linear uncertain system example. The proposed scheme is implemented on a hardware setup of a 2-DOF Helicopter system.

1 Introduction

Many modern systems require accurate tracking of reference commands, continuous control action in order to provide high quality and reliable performance, i.e., robotic manipulator [1], PMSM drive [2], servo system [3, 4] to name just a few. The performance of these systems is getting affected by an unknown external disturbance and parametric uncertainty. The system's parametric uncertainty significantly affects the transient response of a system, and an unknown external disturbance may destabilize the system or causes a steady-state error in the system response. Many techniques have been proposed to control a system affected by an unknown disturbance and parametric uncertainty i.e. Adaptive control [5], Sliding mode control [6, 7], Composite non-linear feedback control, Disturbance observer-based control [8] to mention just a few.

An adaptive control updates the controller parameters based on the system states to stabilize the system and cope with uncertainty and external disturbance. The adaptive law based control has been applied to several practical systems [9, 10]. An unbounded adaptation of the controller parameters may destabilize the system, which is an undesirable effect.

The Sliding mode control (SMC) has gained popularity in the control community due to its robustness against external disturbance and parameter variations. It utilizes a discontinuous control component to mitigate the effect of external disturbance. Many applications have been implemented using Sliding mode control [11,12,13]. The discontinuous control of traditional SMC causes chatter in the system states and may induce wear and tear of the actuator [14].

The composite non-linear feedback control (CNF) consists of linear feedback law and non-linear feedback law. The linear part is designed with a small damping ratio for a speedy response with bounded control input consideration. The non-linear feedback control law adapts the damping of the system based on the tracking error to reduce the overshoot of the system [15]. The application of CNF reported in the literature is limited to a linear system with input saturation [16].

The controllers mentioned above either utilizing a discontinuous control action or large amount of control effort to compensate the effect of parametric uncertainty and an unknown external disturbance. A disturbance observer is an attractive technique to estimate the effect of disturbance without using any additional sensor. Once the disturbance is estimated accurately, it can be compensated in the control action [17]. Disturbance observer-based control has been combined with many control technique [8, 18] to control uncertain systems. In this paper, a finite time disturbance observer (FTDO) [19] is utilized to estimate the lumped disturbance, and it is compensated in a non-linear state feedback control (NLSFC) law.

It is well known that the linear state feedback control is not robust to parametric uncertainty and external disturbances. Further, such controller design requires the exact knowledge of system parameters, which is hardly available in most of the practical systems and ensures asymptotic convergence of the system states. In this paper, an FTDO based NLSFC is designed to provide finite-time convergence of disturbance estimation error. The system states to zero in the presence of parametric uncertainty and external disturbance.

The rest of the paper is organized as follows. The problem formulation is stated for the NLSFC in Sect. 2. The non-linear state feedback control for a second-order nominal system and the uncertain system is described in Sect. 3. The FTDO is derived in Sect. 4 and the generalization of the proposed scheme to \(n-\)th order system is given in Sect. 5. The simulation results of the proposed scheme are shown in Sect. 6 followed by the experimental validation in Sect. 7. The paper is concluded in Sect. 8.

2 Problem formulation

Consider a second order system with following dynamics

$$\begin{aligned} {\dot{x}}_1&= x_2 \nonumber \\ {\dot{x}}_2&= -a_1 x_1 - a_2 x_2 + bu \end{aligned}$$

(1)

where x_1 and x_2 are the system states, *u* is the control input, a_1, a_2 and *b* are the system parameters. The control objective is bring the system states from their initial conditions to origin.

The Linear state feedback control (LSFC) law can be implemented as

$$\begin{aligned} u = -k_1 x_1 - k_2 x_2 \end{aligned}$$

(2)

where, k_1 and k_2 are the state feedback gains to be designed. The state feedback gains can be selected such that the closed system is stable and control objective is achieved. The closed loop dynamics is given by

$$\begin{aligned} {\dot{x}}_1&= x_2 \nonumber \\ {\dot{x}}_2&= -(a_1+k_1) x_1 - (a_2+k_2) x_2 \end{aligned}$$

(3)

or

$$\begin{aligned} \ddot{x}_1 + (a_2 + k_2) {\dot{x}}_1 + (a_1 + k_1) x_1 = 0 \end{aligned}$$

(4)

The LSFC law (2) ensures asymptotic convergence of states of the system (1) and the design of control law requires exact knowledge of system parameters. These limitations are overcome by designing a non-linear state feedback control in the next Section.

3 Non-linear state feedback control law

In this section, first, a Non-linear State Feedback Control is designed for the nominal second-order system (1), and later, it is extended to the system affected by the parametric uncertainty and an unknown external disturbance.

3.1 For nominal system

The proposed NLSFC for the system (1) is given by

$$\begin{aligned} u = \frac{1}{b} \Big [-k_1 \left| x_1 \right| ^{\beta _1} \text {sgn}(x_1) - k_2&\left| x_2 \right| ^{\beta _2} \text {sgn}(x_2) \nonumber \\&+ a_1 x_1 + a_2 x_2\Big] \end{aligned}$$

(5)

where, k_1 and k_2 are the control gains to be designed, the coefficients $\beta _1$ and $\beta _2$ are selected as follow [20]:

$$\begin{aligned} \beta _1&= \beta ,\,\,\, n = 1 \nonumber \\ \beta _{i-1}&= \frac{\beta _i \beta _{i+1}}{2\beta _{i+1}-\beta _i}, \,\, i = 2, 3, 4,..., n \end{aligned}$$

(6)

where $\beta _{n+1} = 1$, $\beta _n = \beta$, $\beta \in (1-\alpha , \,\, 1)$ and $\alpha \in (0,\,\,1).$ The function $\text {sgn}(\cdot)$ is the signum function of any variable. The control gains (k_1 and k_2) are selected such that the all eigenvalues of polynomial $s^2 + k_1 s + k_2$ are in left half side of the complex plane. The closed loop dynamics of system (1) with the proposed control law (5) becomes

$$\begin{aligned} {\dot{x}}_1&= x_2 \nonumber \\ {\dot{x}}_2&= -k_1 \left| x_1 \right| ^{\beta _1} \text {sgn}(x_1) - k_2 \left| x_2 \right| ^{\beta _2} \text {sgn}(x_2) \end{aligned}$$

(7)

If the coefficients $\beta _1$ and $\beta _2$ are selected as per (6) and the control gains are selected such that the polynomial $s^2 + k_1 s + k_2$ then the system states converges to origin from their initial condition in finite time ($t \ge t_c$) [20].

Remark 1

When the coefficients are selected as $\beta _1 = 1$ and $\beta _2 = 1$, the control law becomes Linear State Feedback Control

$$\begin{aligned} u = \frac{1}{b} \Big [(a_1-k_1) x_1 + (a_2 - k_2) x_2 \Big] \end{aligned}$$

(8)

and the system dynamics turns out as

$$\begin{aligned} \ddot{x}_1 + k_2 {\dot{x}}_1 + k_1 x_1 = 0 \end{aligned}$$

(9)

The NLSFC ensures finite time convergence of the system states but it still requires exact knowledge of the system parameters (a_1, a_2 and *b*). This requirement limits the application of the proposed control law to practical systems. In practical systems, the exact value of system parameter may not be known but the partial information may be available i.e. nominal value of parameter is known or the range of parameter variation is known or the upper bounds of parameter is known. Further, the external disturbance acting on the plant may not be measurable or it is expansive to measure the signal. In such situations, the proposed scheme can be implemented as follows.

3.2 In the presence of uncertainty and disturbance

The dynamics of second order uncertain system is given by

$$\begin{aligned} {\dot{x}}_1&= x_2 \nonumber \\ {\dot{x}}_2&= -a_{1n} x_1 - a_{2n} x_2 + b_nu + d({\varvec{x}},t) \end{aligned}$$

(10)

where a_{1n}, a_{2n} and b_n represent the nominal value of the parameters a_1, a_2 and *b* respectively, $d({\varvec{x}},t)$ represents the lumped disturbance acting on the system $d({\varvec{x}},t) = \Delta a_1 x_1 + \Delta a_2 x_2 + \Delta b u + \zeta (t)$, where ${\varvec{x}} = \begin{bmatrix} x_1,&x_2 \end{bmatrix}^T$ and $\zeta (t)$ is an unknown external disturbance acting on the system.

Assumption 1

The lumped disturbance $d({\varvec{x}},t)$ is continuous time unknown function and it is second order differentiable which satisfies following condition

$$\begin{aligned} | \ddot{d}({\varvec{x}},t) | \le L \end{aligned}$$

(11)

where *L* is Lipschitz constant.

For many practical systems, this assumption is realistic. For example, PMSM motor [21], DC–DC converters [22], attitude tracking of rigid aircraft [23], robotic manipulator [24], the load disturbance and the rate of change of load disturbance may change, but the second derivative of disturbance is always bounded.

The control objective is to design a control law for the system (10) such that the system states reach to origin in finite from their initial condition. The proposed control law is now given by

$$\begin{aligned} u = \frac{1}{b_n} \Big [-k_1 \left| x_1 \right| ^{\beta _1} \text {sgn}

(x_1)&- k_2 \left| x_2 \right| ^{\beta _2} \text {sgn}(x_2)\nonumber \\&+ a_{1n} x_1 + a_{2n} x_2\Big] - {\hat{d}} \end{aligned}$$

(12)

where ${\hat{d}}$ is an estimate of lumped disturbance $d({\varvec{x}},t)$. The dynamics of closed loop system becomes

$$\begin{aligned} {\dot{x}}_1&= x_2 \nonumber \\ {\dot{x}}_2&= -k_1 \left| x_1 \right| ^{\beta _1} \text {sgn}(x_1) - k_2 \left| x_2 \right| ^{\beta _2} \text {sgn}(x_2) + {\tilde{d}} \end{aligned}$$

(13)

where, ${\tilde{d}} = d({\varvec{x}},t) - {\hat{d}}$ represents the estimation error of lumped disturbance $d({\varvec{x}},t)$.

Remark 2

It is worth noting that the closed loop dynamics (13) is driven by the disturbance estimation error ${\tilde{d}}$. If it is possible to estimate the lumped disturbance exactly then ${\hat{d}} = d({\varvec{x}},t)$ and ${\tilde{d}} = 0$. The system behaves like nominal system with finite time reach-ability of system states.

3.3 Stability

The stability of closed loop system is derived in the lines of [20] by considering the Lyapunov function as

$$\begin{aligned} V(x_1, x_2) = \frac{1}{2} x_2^2 + \frac{k_1}{\beta _1+1} |x_1 |^{\beta _1+1} \end{aligned}$$

(14)

The time derivative of $V(x_1, x_2)$ can be computed as

$$\begin{aligned} {\dot{V}} = x_2 {\dot{x}}_2 + k_1 x_2 |x_1 |^{\beta _1} \text {sgn}(x_1) \end{aligned}$$

(15)

Simplifying (15) using (13) and (10) as

$$\begin{aligned} {\dot{V}} = -|x_2 |^{\beta _2 +1} + x_2 {\tilde{d}} \end{aligned}$$

(16)

Assuming that the disturbance estimation error ${\tilde{d}}$ goes to zero in a finite time. Thus after a finite time, the time derivative of Lyapunov function is given by,

$$\begin{aligned} {\dot{V}} = -|x_2 |^{\beta _2 +1} \le 0 \end{aligned}$$

(17)

which ensures finite-time convergence of system states (i.e., x_1 and x_2) to zero.

4 Disturbance estimation

It is possible to use any finite time disturbance estimation technique in combination of the proposed scheme. In this paper, we have utilized finite time disturbance observer proposed in [19] to estimate lumped disturbance $d({\varvec{x}},t)$. The dynamics of finite time disturbance observer is given by

$$\begin{aligned} \dot{{\hat{x}}}_2&= -a_{1n} x_1 - a_{2n} x_2 + b_n u + v_0 \nonumber \\ \dot{{\hat{d}}}&= v_1 \nonumber \\ \dot{\hat{{\dot{d}}}}&= -\lambda _2\,\, L\,\, \text {sgn}(\hat{{\dot{d}}} - v_1) \end{aligned}$$

(18)

where, ${\hat{x}}_2$ is the estimate of state x_2, ${\hat{d}}$ is the estimate of lumped disturbance $d({\varvec{x}},t)$ and $\hat{{\dot{d}}}$ is the rate of change of lumped disturbance estimation. The variables v_0 and v_1 are updated as

$$\begin{aligned} v_0&= -\lambda _0 L^{\frac{1}{3}} | {\hat{x}}_2 - x_2|^{\frac{2}{3}} \text {sgn}({\hat{x}}_2 - x_2) + {\hat{d}} \nonumber \\ v_1&= -\lambda _1 L^{\frac{1}{2}} | {\hat{d}} - v_0|^{\frac{1}{2}} \text {sgn}({\hat{d}} - v_0) + \hat{{\dot{d}}} \end{aligned}$$

(19)

where, L is a observer gain and $\lambda _0, \lambda _1$ and $\lambda _2$ are coefficients to be selected by the designer.

Defining the error estimation errors of observer as

$$\begin{aligned} {\tilde{x}}_2&= {\hat{x}}_2 - x_2 \nonumber \\ {\tilde{d}}&= {\hat{d}} - d({\varvec{x}},t) \nonumber \\ \tilde{{\dot{d}}}&= \hat{{\dot{d}}} - {\dot{d}} \end{aligned}$$

(20)

Taking time derivative of (20) to obtain the error dynamics of FTDO

$$\begin{aligned} \dot{{\tilde{x}}}_2&= -\lambda _0 L^{\frac{1}{3}} | {\tilde{x}}_2| ^{\frac{2}{3}} \text {sgn}({\tilde{x}}_2) + {\tilde{d}} \nonumber \\ \dot{{\tilde{d}}}&= -\lambda _1 L^{\frac{1}{2}} | {\tilde{d}} - \dot{{\tilde{x}}}_2|^{\frac{1}{2}} \text {sgn}({\tilde{d}} - \dot{{\tilde{x}}}_2) + \tilde{{\dot{d}}} \nonumber \\ \dot{\tilde{{\dot{d}}}} &\in -\lambda _2\,\, L\,\, \text {sgn}(\dot{{\tilde{d}}} - \dot{\tilde{{\dot{d}}}}) + \begin{bmatrix} -L,\,\, L \end{bmatrix} \end{aligned}$$

(21)

Remark 3

As per Assumption 1, the second derivative of lumped disturbance is bounded. If the observer gain L is selected as $|\ddot{d}({\varvec{x}},t)| \le L$ [19] then the estimation errors go to zero in finite time. Thus, the estimation errors (${\tilde{x}}_2$, ${\tilde{d}}$ and $\dot{{\tilde{d}}}$) go to zero, one can write $x_2 = {\hat{x}}_2$, $d = {\hat{d}}$ and ${\dot{d}} =\hat{{\dot{d}}}$ after finite time ($t \ge t_o$).

Remark 4

The observer gains should be selected in such a way that the finite time of observer is less than the controller time $t_o \le t_c$. Thus, when the system states reach to origin, the performance will not get affected by the lumped disturbance or the disturbance estimation error.

When the disturbance estimation error is not zero (${\tilde{d}}\ne 0$ or $t < t_o$), the closed loop dynamics is governed by

$$\begin{aligned} \ddot{x}_1 + k_2 \left| {\dot{x}}_1 \right| ^{\beta _2} \text {sgn} ({\dot{x}}_1) + k_1 \left| x_1 \right| ^{\beta _1} \text {sgn}(x_1) = {\tilde{d}} \end{aligned}$$

(22)

After $t \ge t_o$,

$$\begin{aligned} \ddot{x}_1 + k_2 \left| {\dot{x}}_1 \right| ^{\beta _2} \text {sgn} ({\dot{x}}_1) + k_1 \left| x_1 \right| ^{\beta _1} \text {sgn}(x_1) = 0 \end{aligned}$$

(23)

5 Generalization to $n-$th order non-linear system

The proposed scheme can be extended to $n-$th order system as follows. A $n-$th order non-linear uncertain system is given by

$$\begin{aligned} {\dot{x}}_1&= x_2 \nonumber \\ {\dot{x}}_2&= x_3 \nonumber \\&\vdots \nonumber \\ {\dot{x}}_{n-1}&= x_n \nonumber \\ {\dot{x}}_n&= a_n({\varvec{x}},t) + b_n \Big (u + d({\varvec{x}},t)\Big) \end{aligned}$$

(24)

where, $a_n({\varvec{x}},t)$ is a known non-linear function and the lumped disturbance is given by $d({\varvec{x}},t) = \Delta a({\varvec{x}},t) + \zeta (t)$. The proposed control law is implemented as follows

$$\begin{aligned} u = \frac{1}{b_n} \Big [a({\varvec{x}},t) - \sum _{i=1}^{n}k_i \left| x_i \right| ^{\beta _i} \text {sgn}(x_i)\Big] - {\hat{d}} \end{aligned}$$

(25)

The estimate of lumped disturbance ${\hat{d}}$ is given by

$$\begin{aligned} \dot{{\hat{x}}}_n&= -a({\varvec{x}},t) + b_n u + v_0 \nonumber \\ \dot{{\hat{d}}}&= v_1 \nonumber \\ \dot{\hat{{\dot{d}}}}&= -\lambda _2\,\, L\,\, \text {sgn}(\hat{{\dot{d}}} - v_1) \end{aligned}$$

(26)

The variables v_0 and v_1 are updated as

$$\begin{aligned} v_0&= -\lambda _0 L^{\frac{1}{3}} | {\hat{x}}_n - x_n|^{\frac{2}{3}} \text {sgn}({\hat{x}}_n - x_n) + {\hat{d}} \nonumber \\ v_1&= -\lambda _1 L^{\frac{1}{2}} | {\hat{d}} - v_0|^{\frac{1}{2}} \text {sgn}({\hat{d}} - v_0) + \hat{{\dot{d}}} \end{aligned}$$

(27)

Remark 5

The proposed scheme indeed utilizes discontinuous components in the higher derivatives of the control action. The proposed scheme employs a finite time disturbance observer to estimate an unknown disturbance and parametric uncertainty accurately, which avoids excessive control input compared to the traditional discontinuous controller (i.e., Sliding Mode Controller). Thus, it avoids high power control action to mitigate the effect of lumped disturbance.

The closed loop dynamics is governed in case of $t < t_o$ by

$$\begin{aligned} {x}_1^{(n)} + \sum _{i=1}^{n} k_i \left| {x}_1^{(i-1)} \right| ^{\beta _i} \text {sgn}({x}_1^{(i-1)}) = {\tilde{d}} \end{aligned}$$

(28)

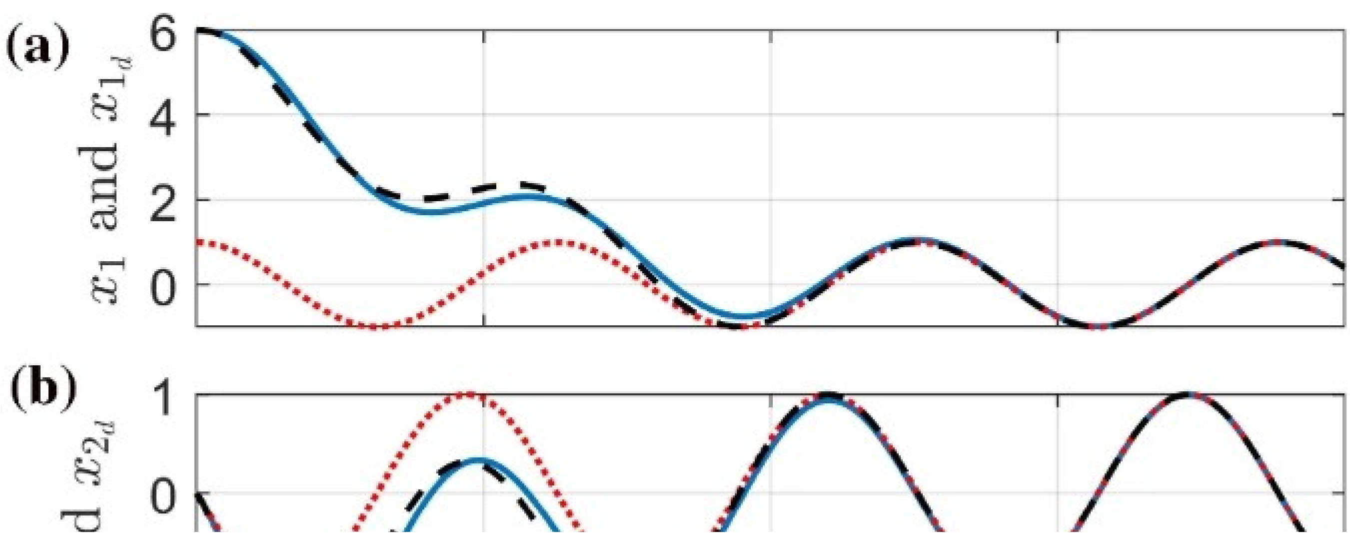
After $t \ge t_o$,

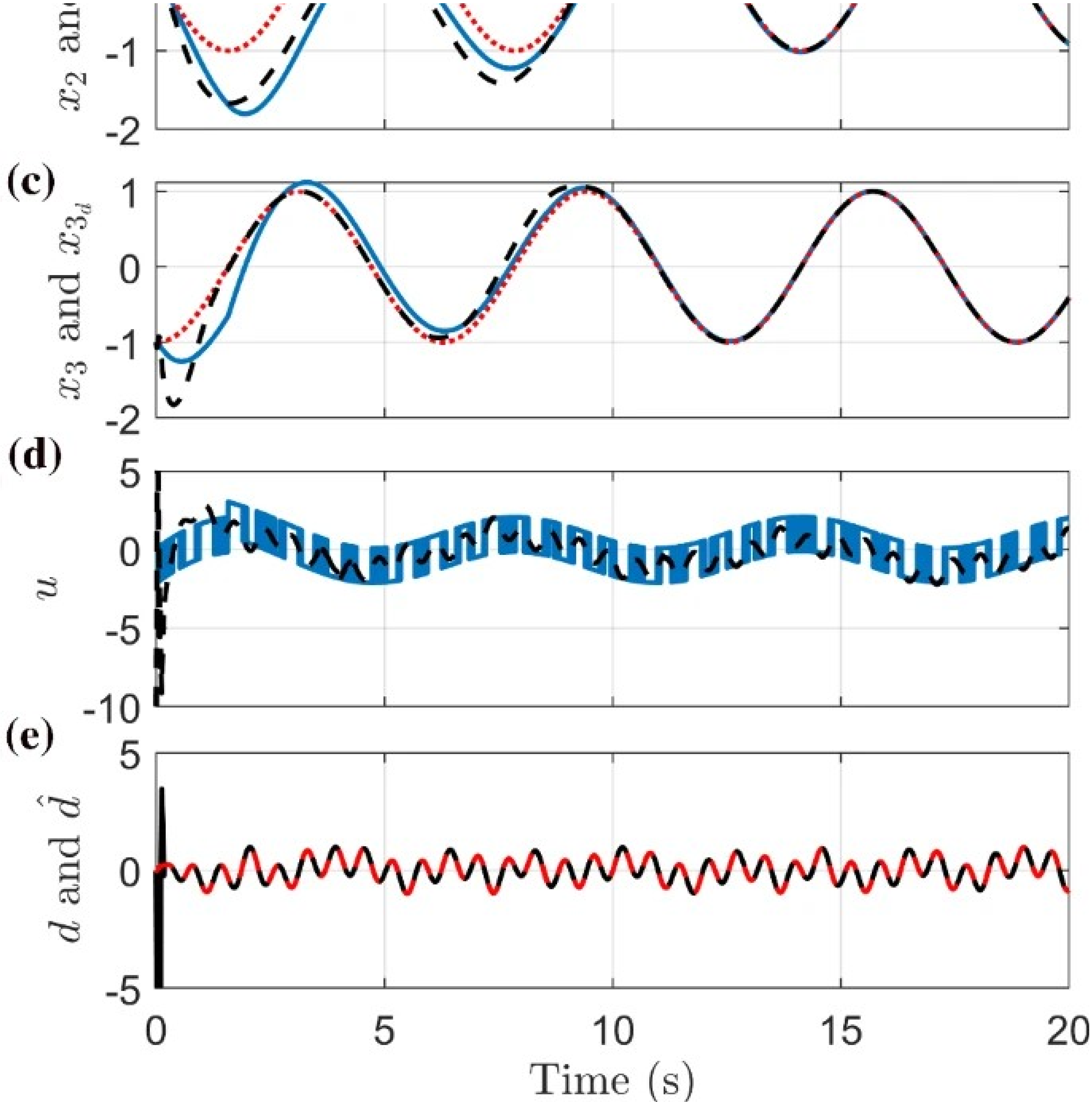
$$\begin{aligned} {x}_1^{(n)} + \sum _{i=1}^{n} k_i \left| {x}_1^{(i-1)} \right| ^{\beta _i} \text {sgn}({x}_1^{(i-1)}) = 0 \end{aligned}$$

(29)

6 Simulation results

Fig. 1





Comparative simulation results with the proposed scheme (dashed) and the SMC (solid): a \(x_1\) and \(x_{1_d}\) (dotted), b \(x_2\) and \(x_{2_d}\) (dotted), c \(x_3\) and \(x_{3_d}\) (dotted), d *u*, e *d* (dotted) and \({\hat{d}}\) (solid)

To verify the proposed scheme's effectiveness, it is applied to a third-order nonlinear system subjected to an external disturbance and parametric uncertainty. The proposed scheme's performance is compared with a sliding mode control reported in [25]. The dynamics of a third-order nonlinear uncertain system is given by [25],

$$\begin{aligned} {\dot{x}}_1&= x_2 \nonumber \\ {\dot{x}}_2&= x_3 \nonumber \\ {\dot{x}}_3&= b \Big (u + d({\varvec{x}},t) \Big) \end{aligned}$$

(30)

where $d({\varvec{x}},t) = 0.39 \sin (x_1 x_2 + x_3 \sqrt{t}) + 0.6 \sin (10t)$ consists of parametric uncertainty and the external disturbance acting on the plant and $b = 1$. The initial condition of plant is considered as $x(0) = \begin{bmatrix} 6, \,\, 0,\,\, -1 \end{bmatrix}^T$ and the reference trajectories are selected as $\begin{bmatrix} x_{1_d}, \,\, x_{2_d}, \,\, x_{3_d} \end{bmatrix}$ $= \begin{bmatrix} -\sin (t-1.57), \,\, -\cos (t-1.57), \,\, \sin (t-1.57) \end{bmatrix}$. The proposed control law is implemented as

$$\begin{aligned} u =&-k_1 \left| x_1-x_{1_d} \right| ^{\beta _1} \text {sgn}(x_1-x_{1_d}) \nonumber \\&- k_2 \left| x_2-x_{2_d} \right| ^{\beta _2} \text {sgn}(x_2-x_{2_d}) \nonumber \\&- k_3 \left| x_3-x_{3_d} \right| ^{\beta _3} \text {sgn}(x_3-x_{3_d}) + {\dot{x}}_{3_d} - {\hat{d}} \end{aligned}$$

(31)

Table 1 Nominal parameters of hardware setup [26]

where the control parameters are selected as $k_1 = 5, k_2 = 9, k_3 = 5$, $\beta _1 = \frac{5}{11}, \beta _2 = \frac{5}{15}$ and $\beta _3 = \frac{5}{19}$ The finite time disturbance observer is implemented as

$$\begin{aligned} \dot{{\hat{x}}}_3&= u + v_0 \nonumber \\ \dot{{\hat{d}}}&= v_1 \nonumber \\ \dot{\hat{{\dot{d}}}}&= -\lambda _2\cdot L\,\, \text {sgn}(\hat{{\dot{d}}} - v_1) \end{aligned}$$

(32)

The variables v_0 and v_1 are updated as

$$\begin{aligned} v_0&= -\lambda _0 \cdot L^{\frac{1}{3}} | {\hat{x}}_3 - x_3|^{\frac{2}{3}} \text {sgn}({\hat{x}}_3 - x_3) + {\hat{d}} \nonumber \\ v_1&= -\lambda _1\cdot L^{\frac{1}{2}} | {\hat{d}} - v_0|^{\frac{1}{2}} \text {sgn}({\hat{d}} - v_0) + \hat{{\dot{d}}} \end{aligned}$$

(33)

where the observer parameters are selected as $\lambda _0 = 3, \lambda _1 = 2.5, \lambda _2 = 2,$ and $L = 1500.$ The sliding mode controller reported in [25] is implemented as follows. The sliding surface (s) is selected as

$$\begin{aligned} s = c_1 (x_1-x_{1_d}) + c_2 (x_2-x_{2_d}) + x_3-x_{3_d} +&A + Bt; \nonumber \\&t \le t_f \nonumber \\ s = c_1 (x_1-x_{1_d}) + c_2 (x_2-x_{2_d}) + x_3-x_{3_d}; \,\,&\text {otherwise}. \end{aligned}$$

(34)

where the value of coefficients is selected as $c_1 = 0.31,$ $c_2 = 1.12,\,\, A = -1.57,\,\, B = 1,\,\, t_f = 1.6$ s. The control law is implemented as

$$\begin{aligned} u = -c_2(x_3-x_{3_d}) - c_1(x_2-x_{2_d}) + {\dot{x}}_{3_d} - B - \gamma \text {sgn} (s) \end{aligned}$$

(35)

where the switching gain is selected as $\gamma = 1.1$. The comparative plots of the proposed scheme and the SMC are shown in Fig. 1. It can be observed that the tracking error $x_1 - x_{1_d}$ goes to zero around $t = 9$ s with the proposed scheme and $t = 11$ s with the SMC as shown in Fig. 1a. It is worth noting that the SMC brings the sliding surface to zero in finite time, which leads the system dynamics to

$$\begin{aligned} \ddot{x}_1 - \ddot{x}_{1d} + c_2 ({\dot{x}}_1 - {\dot{x}}_{1d}) + c_1 ({x}_1 - {x}_{1d}) = 0 \end{aligned}$$

(36)

Thus, the tracking error $x_1 - x_{1d}$ goes to zero asymptotically with the SMC. In the case of the proposed scheme, the tracking error goes to zero in a finite time. The comparative plot of the control inputs is shown in Fig. 1d. The control input with the SMC is discontinuous as it utilizes switching term and the control input with the proposed scheme is continuous and free from chattering as it uses FTDO to estimate the lumped disturbance. Further, in case of SMC the magnitude of discontinuous control is depend on the selection of switching gain γ, which need to be selected as $\gamma > \left| d({\varvec{x}},t) \right| _{\text {max}}$. If $\left| d({\varvec{x}},t) \right| _{\text {max}}$ is not known to the designer, requires larger value of γ and it may cause higher discontinuous control effort. On the other hand, the proposed scheme utilizes FTDO to estimate the exact value of $d({\varvec{x}},t)$ as shown in Fig. 1e, which generates smooth control action.

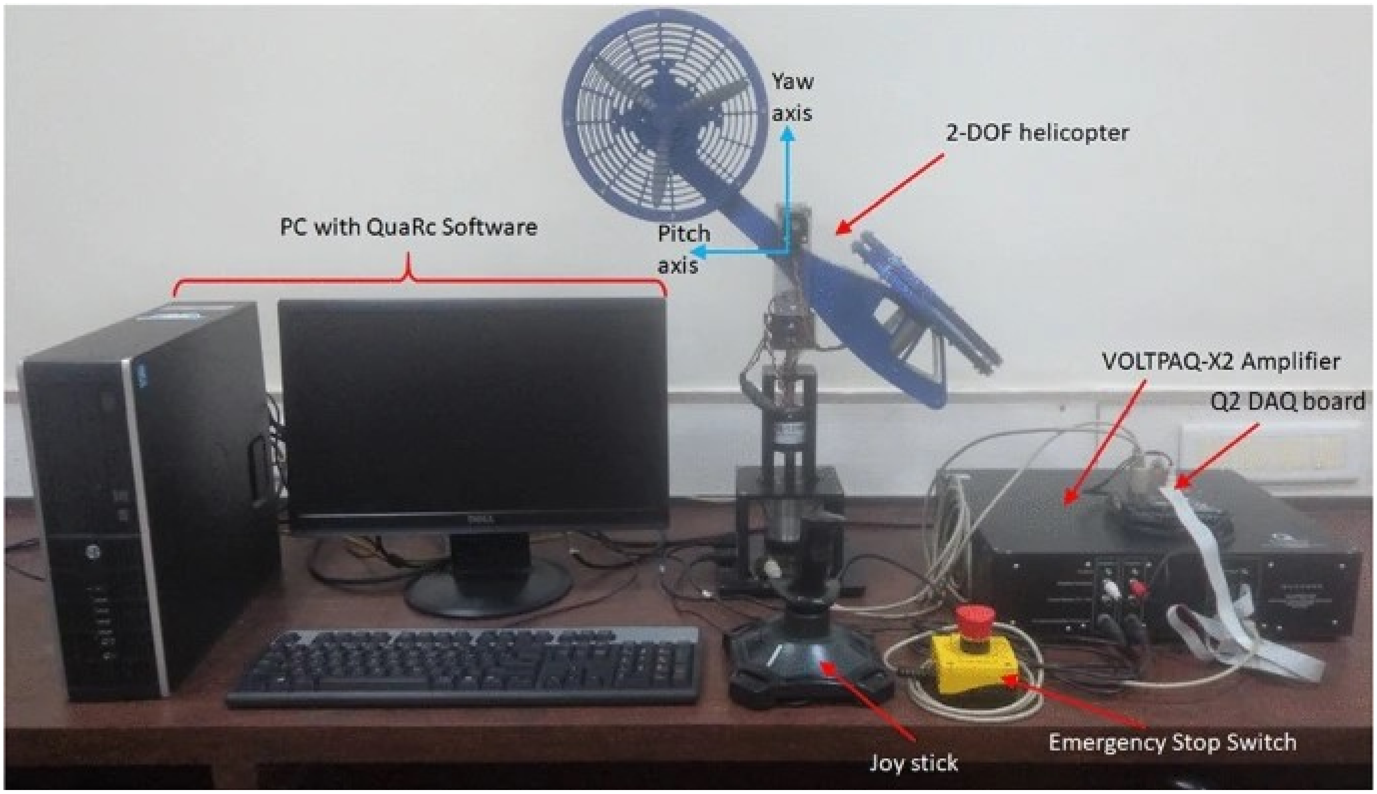
7 Experimental validation

In order to demonstrate the effectiveness of the proposed scheme, it has been applied to a nonlinear plant of a 2-DOF Helicopter system [26].

7.1 Hardware setup

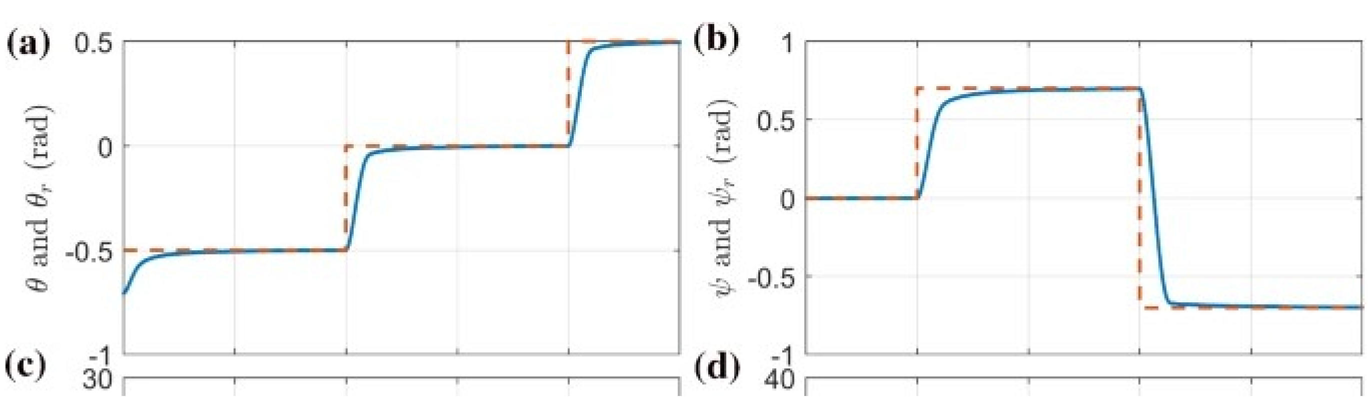
The experimental setup consists of a 2-DOF Helicopter plant with a VOLTPAQ-X2 power amplifier, Q2 DAQ board, Emergency stop switch, Logitech joystick, and personal computer installed with QuaRc and MATLAB tools. The VOLTPAQ-X2 amplifies the controller commands and supply to the plant. The Q2 DAQ board receives plant feedback signals and the control commands from the QuaRc software and sends the control commands to the VOLTPAQ-X2 amplifier. The reference signals are either generated by a Simulink block or Logitech joystick. The emergency stop switch disables all operations in case of any emergency. The QuaRc tool provides an interface between Simulink and Q2 DAQ board. The hardware setup consists of two brush-less motors to control the pitch angle (θ) and the yaw angle (ψ). The pitch and yaw angles are measured using 10 bits optical encoders, which send feedback signals to the Q2 DAQ board (Fig. 2).

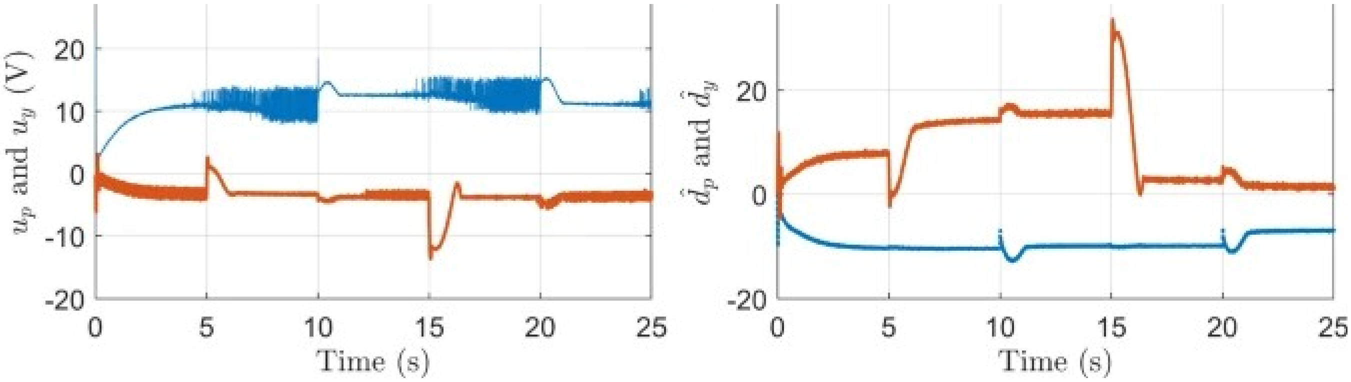
Fig. 2



Hardware setup of 2-DOF Helicopter system

Fig. 3





Experimental results: a pitch angle θ (solid) and $\theta _r$ (dashed), b yaw angle ψ (solid) and $\psi _r$ (dashed), c control inputs u_y (solid) and u_p (dashed), d disturbance estimation ${\hat{d}}_y$ (solid) and ${\hat{d}}_p$ (dashed)

7.2 Plant dynamics

The dynamics of the nonlinear 2-DOF helicopter system is considered as [26],

$$\begin{aligned} \ddot{\theta }&= - \frac{B_\theta {\dot{\theta }} + m l^2{\dot{\psi }}^2 \sin \theta \cos \theta + mgl\cos \theta }{J_\theta + m l^2} \nonumber \\&\quad + \frac{k_{\theta \theta }}{J_\theta + m l^2} u_\theta + \frac{k_{\theta \psi }}{J_\theta + m l^2} u_\psi \end{aligned}$$

(37)

$$\begin{aligned} \ddot{\psi }&= \frac{2m l^2 {\dot{\psi }} {\dot{\theta }} \sin \theta \cos \theta }{J_\psi + ml^2 \cos ^2\theta } - \frac{B_\psi {\dot{\psi }}}{J_\psi + ml^2 \cos ^2\theta } \nonumber \\&\quad + \frac{k_{\psi \theta }}{J_\psi + ml^2 \cos ^2\theta } u_\theta + \frac{k_{\psi \psi }}{J_\psi + ml^2 \cos ^2\theta } u_\psi \end{aligned}$$

(38)

where u_θ and u_ψ are the control inputs to be designed. The objective is to design the control inputs to bring the difference between actual angles, rate of change of actual angles and the reference trajectories $e_{1\theta } = \theta - \theta _r$, $e_{2\theta } = {\dot{\theta }} - {\dot{\theta }}_r$, $e_{1\psi } = \psi - \psi _r$ and $e_{2\psi } =$

${\dot{\psi }} - {\dot{\psi }}_r$ to zero respectively in the presence of parametric uncertainty, unknown non-linearity and a coupling between two inputs. The nominal parameters considered for the experimental validation are given in Table 1.

7.3 Control law

The control inputs u_θ and u_ψ are designed by considering two independent sub-systems of pitch and yaw dynamics. The coupling between two sub-systems, the parametric uncertainty and the non-linearity, is considered the lumped disturbances (d_θ and d_ψ) acting on the sub-systems. The control inputs are selected as

$$\begin{aligned} u_\theta&= \frac{1}{b_{n\theta }} \Big [a_\theta (\theta , t) - \sum _{i=1}^{2}k_{i\theta } \left| e_{i\theta } \right| ^{\beta _{i\theta }} \text {sgn}(e_{i\theta })\Big] - {\hat{d}}_\theta \end{aligned}$$

(39)

$$\begin{aligned} u_\psi&= \frac{1}{b_{n\psi }} \Big [a_\psi (\psi ,t) - \sum _{i=1}^{2} k_{i\psi } \left| e_{i\psi } \right| ^{\beta _{i\psi }} \text {sgn}(e_{i\psi })\Big] - {\hat{d}}_\psi \end{aligned}$$

(40)

where the lumped disturbances ${\hat{d}}_\theta$ and ${\hat{d}}_\psi$ are implemented as per (26) and (27). The control and observer parameters selected for experimental validation are given in Table 2. The known functions $a_\theta (\theta ,t) = -3.50\,\theta$ and $a_\psi (\psi ,t) = -9.28\,\psi$.

Table 2 The control parameters and observer parameters used for experimental validation

7.4 Results

Figure 3 shows the experimental results of the proposed scheme. The tracking of pitch and yaw reference signals are shown in Fig. 3a and b, respectively. One can observe the pitch and yaw angles track the reference signals very well. Thus the proposed scheme brings the tracking errors to zero from their initial conditions. The control inputs are getting adjusted as the reference signals change, as shown in Fig. 3c. The plot of lumped disturbance estimation is shown in Fig. 3d. One can see that the proposed scheme accurately estimates the system nonlinearity, uncertainty, and the coupling between two sub-systems as the lumped disturbances. Thus the proposed scheme was successfully implemented for a non-linear uncertain system.

8 Conclusion

In this paper, a finite time disturbance observer-based non-linear state feedback control law is proposed for an uncertain system. The results are generalized to \(n-\)th order non-linear uncertain system. The proposed observer-controller combination ensures finite time convergence of the disturbance estimation error and the tracking errors of system states to zero in the presence of system non-linearity, parametric uncertainty, and an unknown external disturbance. The proposed scheme's performance is compared with a well-known sliding mode controller, and it shows better performance in terms of smoothness of the control action and the tracking of the reference trajectory. Further, the proposed scheme is implemented on a hardware setup of a 2-DOF Helicopter system in a laboratory.

References

1. Gambhire S, Kanth KS, Malvatkar G, Londhe P (2019) Robust fast finite-time sliding mode control for industrial robot manipulators. Int J Dyn Control 7(2):607–618

Article MathSciNet Google Scholar

2. Yang J, Chen W, Li S, Guo L, Yan Y (2017) Disturbance/uncertainty estimation and attenuation techniques in pmsm drives: a survey. IEEE Trans Ind Electron 64(4):3273–3285

Article Google Scholar

3. She JH, Ohyama Y, Nakano M (2005) A new approach to the estimation and rejection of disturbances in servo systems. IEEE Trans Control Syst Technol 13(3):378–385

Article Google Scholar

4. Ginoya D, Gutte CM, Shendge P, Phadke S (2016) State-and-disturbance-observer-based sliding mode control of magnetic levitation systems. Trans Inst Meas Control 38(6):751–763

Article Google Scholar

5. Ioannou PA, Sun J (2012) Robust adaptive control. Courier Corporation

6. Young KD, Utkin VI, Ozguner U (1999) A control engineer's guide to sliding mode control. IEEE Trans Control Syst Technol 7(3):328–342

Article Google Scholar

7. Utkin V, Guldner J, Shi J (2017) Sliding mode control in electro-mechanical systems. CRC Press, Boca Raton

Book Google Scholar

8. Chen WH, Yang J, Guo L, Li S (2015) Disturbance-observer-based control and related methods-an overview. IEEE Trans Ind Electron 63(2):1083–1095

Article Google Scholar

9. Sastry SS, Isidori A (1989) Adaptive control of linearizable systems. IEEE Trans Autom

Control 34(11):1123–1131

Article MathSciNet Google Scholar

10. Åström KJ (1983) Theory and applications of adaptive control-a survey. Automatica 19(5):471–486

Article Google Scholar

11. Utkin VI (1993) Sliding mode control design principles and applications to electric drives. IEEE Trans Ind Electron 40(1):23–36

Article Google Scholar

12. Azar AT, Zhu Q (2015) Advances and applications in sliding mode control systems. Springer, Berlin

Book Google Scholar

13. Feng Y, Han F, Yu X (2014) Chattering free full-order sliding-mode control. Automatica 50(4):1310–1314

Article MathSciNet Google Scholar

14. Utkin V, Lee H (2006) Chattering problem in sliding mode control systems. In: International workshop on variable structure systems, 2006. VSS'06., pp 346–350, IEEE

15. Chen BM, Lee TH, Peng K, Venkataramanan V (2003) Composite nonlinear feedback control for linear systems with input saturation: theory and an application. IEEE Trans Autom Control 48(3):427–439

Article MathSciNet Google Scholar

16. Venkataramanan V, Peng K, Chen BM, Lee TH (2003) Discrete-time composite nonlinear feedback control with an application in design of a hard disk drive servo system. IEEE Trans Control Syst Technol 11(1):16–23

Article Google Scholar

17. Chen WH (2004) Disturbance observer based control for nonlinear systems. IEEE/ASME Trans Mechatron 9(4):706–710

Article Google Scholar

18. Ginoya D, Shendge P, Phadke S (2013) Sliding mode control for mismatched uncertain systems using an extended disturbance observer. IEEE Trans Ind Electron 61(4):1983–1992

Article Google Scholar

19. Shtessel YB, Shkolnikov IA, Levant A (2007) Smooth second-order sliding modes: missile guidance application. Automatica 43(8):1470–1476

Article MathSciNet Google Scholar

20. Bhat SP, Bernstein DS (1997) Finite-time stability of homogeneous systems. In: Proceedings of the 1997 American control conference, vol 4, pp 2513–2514, IEEE

21. Yang J, Li S, Su J, Yu X (2013) Continuous nonsingular terminal sliding mode control for

systems with mismatched disturbances. Automatica 49(7):2287–2291

Article MathSciNet Google Scholar

22. Wang J, Li S, Yang J, Wu B, Li Q (2016) Finite-time disturbance observer based non-singular terminal sliding-mode control for pulse width modulation based DC–DC buck converters with mismatched load disturbances. IET Power Electron 9(9):1995–2002

Article Google Scholar

23. Lan Q, Qian C, Li S (2017) Finite-time disturbance observer design and attitude tracking control of a rigid spacecraft. J Dyn Syst Meas Control 139(6)

24. Cao P, Gan Y, Dai X (2019) Finite-time disturbance observer for robotic manipulators. Sensors 19(8):1943

Article Google Scholar

25. Bartoszewicz A, Nowacka A (2007) Sliding-mode control of the third-order system subject to velocity, acceleration and input signal constraints. Int J Adapt Control Signal Process 21(8–9):779–794

Article MathSciNet Google Scholar

26. Quanser: 2-DOF helicopter user and control manuals (Markham, Ontario, 2006)

Author information

Authors and Affiliations

Dr. Babasaheb Ambedkar Marathwada University, Aurangabad, India

Ajay Borkar

Jagadamba Eduction Society Nashik's S. N. D. College of Engineering and Research, Yeola, Nashik, India

P. M. Patil

Corresponding author

Correspondence to Ajay Borkar.

Ethics declarations

Conflict of interest

The authors declare that they have no conflict of interest

Rights and permissions

About this article

Cite this article

Borkar, A., Patil, P.M. Non-linear state feedback control for uncertain systems using a finite time disturbance observer. *Int. J. Dynam. Control* 10, 427–434 (2022). https://doi.org/10.1007/s40435-021-00817-0

Received	Revised	Accepted
30 November 2020	12 May 2021	17 May 2021

Published	Issue Date
26 May 2021	April 2022

DOI

https://doi.org/10.1007/s40435-021-00817-0

Share this article

Anyone you share the following link with will be able to read this content:

Get shareable link

Provided by the Springer Nature SharedIt content-sharing initiative

Keywords

Non-linear state feedback control

Finite time disturbance observer

Non-linear control