

Super twisting observer based full order sliding mode control

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Abstract

In this paper, a super twisting observer-based full order sliding mode control is proposed for a non-linear uncertain system. The super twisting observer estimates an $(n-1)$ th system state and the derivative of the $(n-1)$ th system state. It ensures finite convergence of estimation error to zero. The proposed method retains attractive features of full order sliding mode control like finite-time convergence of system states, continuous control, and the

system's full order dynamics when the system is in the sliding mode. The scheme is validated on a 2-DOF helicopter system laboratory setup, and the performance is compared with two well-known observers, i.e., a non-linear extended state observer and a sliding mode observer-based control using two-link manipulator example.

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1 Introduction

Sliding Mode Control (SMC) has gained popularity in many fields due to its robust performance in the presence of external disturbances and parametric uncertainty [1]. The traditional SMC utilizes discontinuous control, and the system behaves as a reduced-order system when the system is in sliding mode. It ensures finite-time convergence of sliding surface, but the system states converge to zero asymptotically.

Many researches have been carried to reduce the chattering effect in the literature [2, 3]. Among several methodologies present in the literature, Super Twisting Control became popular because of continuous control, finite-time convergence of sliding variable, and its derivative [4]. Many applications were constructed using a super twisting controller, such as n -link manipulator [5], biotechnological process [6], Unmanned Aerial Vehicles [7], wind turbines [8], permanent magnet synchronous linear motor [9], induction generator [10] to name just a few. In this paper, an observer is designed to estimate the derivative of an $(n-1)$ th state of an $(n-1)$ th order system, which utilizes the Super Twisting algorithm for the estimation.

The restriction of reduced-order system dynamics and/ or asymptotic convergence of system states is overcome by some techniques i.e. Terminal Sliding Mode Control [11], Higher-Order Sliding Mode control [12, 13], Full Order Sliding Mode Control [14, 15]. Terminal Sliding Mode Control (TSMC) utilizes a non-linear sliding surface, which ensures finite convergence of system states, fast response, and higher precision. The convergence of system states becomes slower when they are near the origin, and the control is singular in nature at the origin [16].

Higher-Order Sliding Mode Control (HOSMC) ensures finite-time convergence of sliding variables and their derivatives with a continuous-time control input. The performance of HOSMC deteriorates in the presence of unmodeled dynamics and requires relative degree one [17].

Initially, Full Order Sliding Mode Control (FOSMC) was proposed to overcome the limitation of reduced-order dynamics. The FOSMC proposed in [14] eliminates the chattering issue, and it has attractive features like finite-time convergence of sliding variable, system states, and the dynamics of the system is full order when the system is in sliding mode. However, for an $(n-1)$ th order system, the implementation of FOSMC requires all system states (x_1, x_2, \dots, x_n) and the derivative of state x_n (i.e. \dot{x}_n). This requires an additional sensor for the measurement of signal \dot{x}_n ; it may not be feasible in many practical systems. In this paper, the FOSMC is made implementable using only system states (x_1, x_2, \dots, x_n) . The Super Twisting Observer (STO) is proposed to estimate the derivative of x_n without using any additional sensors. The STO ensures finite-time convergence of estimation errors simultaneously; it retains all attractive features of FOSMC. The Super Twisting Observer reported in the literature either estimates the system state(s) and/or estimates an unknown disturbance acting on the system [4, 18, 19]. In contrast to the reported applications, a super twisting observer is proposed to estimate the derivative of the system state in this paper.

The rest of the paper is organized as follows: Sect. 2 describes the problem statement for an $(n-1)$ th order non-linear uncertain system. The super twisting observer-based full order sliding mode control is derived in Sect. 3. The performance of proposed scheme is compared with a non-linear extended state observer and a sliding mode observer based control in Sect. 4 and experimental validation of the proposed scheme is shown in Sect. 5 and followed by the conclusion in Sect. 6.

2 Problem statement

2.1 System dynamics

Consider $(n-1)$ th order non-linear uncertain system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= f(\vec{x}, t) + b \left(u + \bar{d}(\vec{x}, t) \right) \end{aligned}$$

(1)

where, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}^T$ is the system states, the function $f(\vec{x}, t)$ is known continuous-time function, b is the known input parameter of the system and u is the control input to be designed.

Assumption 1

The term $\bar{d}(\vec{x}, t)$ is a partial known function, which represents the parametric uncertainty and unknown external disturbances. It satisfies following condition:

$$\left| \bar{d}(\vec{x}, t) \right| \leq k_d$$

(2)

where $(k_d > 0)$ is a bounded constant.

Assumption 2

The derivative of disturbance $\bar{d}(\vec{x}, t)$ satisfies following condition:

$$\left| \dot{\bar{d}}(\vec{x}, t) \right| < \gamma_d$$

(3)

where (γ_d) is a bounded constant.

The objective of control law is to design a full order sliding mode control for the system (1) and established the sliding motion in finite time in the presence of unknown external disturbances.

2.2 Control law

In order to show the need of super twisting observer initially full order sliding mode control [14] is explained by considering that the derivative of $(n-1)$ th state is available for the design of control law. It is observed that the implementation FOSMC requires the system states as well as the derivative of $(n-1)$ th state. In order to accomplish the need of additional signal (i.e., the derivative of $(n-1)$ th state) for an implementation of FOSMC, a super twisting observer is designed. Defining the sliding variable for the system (1) as:

$$s = \dot{x}_n + \sum_{i=1}^n c_i \text{sgn}(x_i) |x_i|^{\alpha_i} \quad (4)$$

(4)

where the coefficients $(c_i > 0)$ are selected such that the polynomial $(s^{n-1} + \sum_{i=0}^{n-1} c_{n-i} s^{n-i-1})$ is Hurwitz. The coefficients (α_i) are selected as per [14].

When the sliding mode is established $(s = 0)$ the system will behave as

$$\dot{x}_n + \sum_{i=1}^n c_i \text{sgn}(x_i) |x_i|^{\alpha_i} = 0 \quad (5)$$

(5)

The control input u is selected as:

$$u = \frac{1}{b} \left(u_{eq} + u_n \right) \quad (6)$$

(6)

where $(b \neq 0)$. The control input consists two parts: the equivalent control input (u_{eq}) and the disturbance compensation term (u_n) . The equivalent control input (u_{eq}) is selected as:

$$\begin{aligned} u_{eq} = -f(\vec{x}, t) - \sum_{i=0}^n c_i | \text{sgn}(x_i) | \left| x_i \right|^{\alpha_i} \end{aligned}$$

(7)

The disturbance compensation control input is designed as follows:

$$\begin{aligned} \dot{u}_n + \lambda u_n = v \quad v = -\left(k_d + \lambda + \gamma_d + \delta \right) \text{sgn}(s) \end{aligned}$$

(8)

where, the control gains are selected as $(\lambda > 0)$ and $(\delta > 0)$.

Remark 1

The sliding variable s will reach to zero in finite time and similarly the system states (x_1, x_2, \dots, x_n) will also converge to zero in finite time [14].

Remark 2

If the system states (x_1, x_2, \dots, x_n) and the derivative of state (x_n) (i.e. \dot{x}_n) are available for the implementation, one can apply the FOSMC to system (1) as shown in [14]. For many practical system, the measurement of system states is available but the measurement of derivative of state may not be possible because of an unavailability of sensor or the solution is very expensive.

In this paper, a Super Twisting Observer is employed to estimate the derivative of $(n\text{-th})$ system state of an $(n\text{-th})$ order system. It is shown in the paper that when the STO based FOSMC is applied to system (1), the estimation of derivative of system state and the convergence of sliding variable can be achieved.

3 Super twisting observer based control

In this section, first a super twisting observer is presented to estimate an $(n-1)$ th state and the derivative of $(n-1)$ th state of an $(n-1)$ th order non-linear uncertain system and then the full order sliding mode control law based on a STO is derived.

3.1 STO

The dynamics of super twisting observer to estimate an $(n-1)$ th system state $(z_1 = x_{n-1})$ and the derivative of $(n-1)$ th system state $(z_2 = \dot{x}_{n-1})$ of (1) is given by

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 + k_1 |z_1 - \hat{z}_1|^{\frac{1}{2}} \text{sgn}(z_1 - \hat{z}_1) \\ \dot{\hat{z}}_2 &= k_2 |z_1 - \hat{z}_1| \end{aligned} \quad (9)$$

where, the (k_1) and (k_2) are the observer gains to be designed, (\hat{z}_1) is the estimate of $(n-1)$ th system state (x_{n-1}) $(= z_1)$ and (\hat{z}_2) is the estimate of derivative of $(n-1)$ th system state (\dot{x}_{n-1}) $(= z_2)$.

Let the estimation error defined as $(\tilde{z}_1 = z_1 - \hat{z}_1)$ $(\tilde{z}_2 = z_2 - \hat{z}_2)$. The dynamics of observer can be written as

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 + k_1 |\tilde{z}_1|^{\frac{1}{2}} \text{sgn}(\tilde{z}_1) \\ \dot{\hat{z}}_2 &= k_2 |\tilde{z}_1| \end{aligned} \quad (10)$$

The error dynamics of STO calculated using (1) and (10) is given by

$$\begin{aligned} \dot{\tilde{z}}_1 &= \tilde{z}_2 - k_1 |\tilde{z}_1|^{\frac{1}{2}} \text{sgn}(\tilde{z}_1) \\ \dot{\tilde{z}}_2 &= \frac{\partial f(\vec{x}, t)}{\partial \vec{x}} \bar{d}(\vec{x}, t) + \dot{\bar{d}}(\vec{x}, t) + b \dot{u} - k_2 \text{sgn}(\tilde{z}_1) \end{aligned}$$

(11)

Remark 3

As per Assumption 2, the derivative of disturbance is bounded. If the observer gains are selected as $(k_1 \geq 1.5 \sqrt{\beta_d})$ and $(k_2 \geq 1.1\beta_d)$ (where $\beta_d = \max_{t \in [0, \infty)} \left\| \frac{\partial f(\vec{x}, t)}{\partial \vec{x}} \right\| + \|\dot{u}\| + \gamma_d$) then the estimation errors go to zero in finite time. Thus, the estimation errors go to zero, one can write $(x_n = \hat{z}_1)$ and $(\dot{x}_n = \hat{z}_2)$ after finite time.

As the estimation errors (\tilde{z}_1) and (\tilde{z}_2) satisfy the differential inclusion

$$\begin{aligned} \dot{\tilde{z}}_1 &= \tilde{z}_2 - k_1 |\tilde{z}_1|^{\frac{1}{2}} \text{sgn}(\tilde{z}_1) \\ \dot{\tilde{z}}_2 &\in [\beta_d^+, \beta_d^-] - k_2 \text{sgn}(\tilde{z}_1) \end{aligned}$$

(12)

The derivative of $(\dot{\tilde{z}}_1)$ can be calculated as

$$\begin{aligned} \ddot{\tilde{z}}_1 &\in [\beta_d^+, \beta_d^-] - \frac{k_1}{2} |\tilde{z}_1|^{-1/2} - k_2 \text{sgn}(\tilde{z}_1) \end{aligned}$$

(13)

The finite time convergence of (\tilde{z}_1) can be proved in the sense of Fillipov's differential inclusions [20]. When the estimation error goes to zero $(\tilde{z}_1 = 0)$ then one can write $(\tilde{z}_2 = \dot{\tilde{z}}_1)$. The dynamics of (\tilde{z}_2) is given by

$$\begin{aligned} \dot{\tilde{z}}_2 &= [\beta_d^+, \beta_d^-] - k_2 \text{sgn}(\tilde{z}_1) \end{aligned}$$

(14)

Thus, the estimation error (\tilde{z}_2) is bounded in a small vicinity of the origin i.e.

$$\begin{aligned} 0 < k_2 - \beta_d^- \leq |\dot{\tilde{z}}_2| \leq k_2 + \beta_d^+ \end{aligned}$$

(15)

Thus, the estimation errors (\tilde{z}_1) and (\tilde{z}_2) converge to zero in finite-time.

3.2 FOSMC

The STO based FOSMC can be implemented as follow. The sliding surface (\hat{s}) is given by

$$\begin{aligned} \hat{s} = \hat{\dot{x}}_n + c_n \text{sgn}(\hat{x}_n) |\hat{x}_n|^\alpha_n + \sum_{i=1}^{n-1} c_i \text{sgn}(x_i) |x_i|^{\alpha_i} \end{aligned}$$

(16)

The control law u can be implemented as

$$u = \frac{1}{b} \Big(u_{\text{eq}} + u_n \Big)$$

(17)

where, the equivalent control (u_{eq}) and the disturbance compensation control (u_n) are now updated as:

$$\begin{aligned} u_{\text{eq}} = -f(\vec{x}, t) - c_n \text{sgn}(\hat{x}_n) |\hat{x}_n|^\alpha_n - \sum_{i=1}^{n-1} c_i \text{sgn}(x_i) |x_i|^{\alpha_i} \\ \dot{u}_n + \lambda u_n = v \quad \&v = -\Big(k_d + \lambda \gamma_d + \delta \Big) \text{sgn}(\hat{s}) \end{aligned}$$

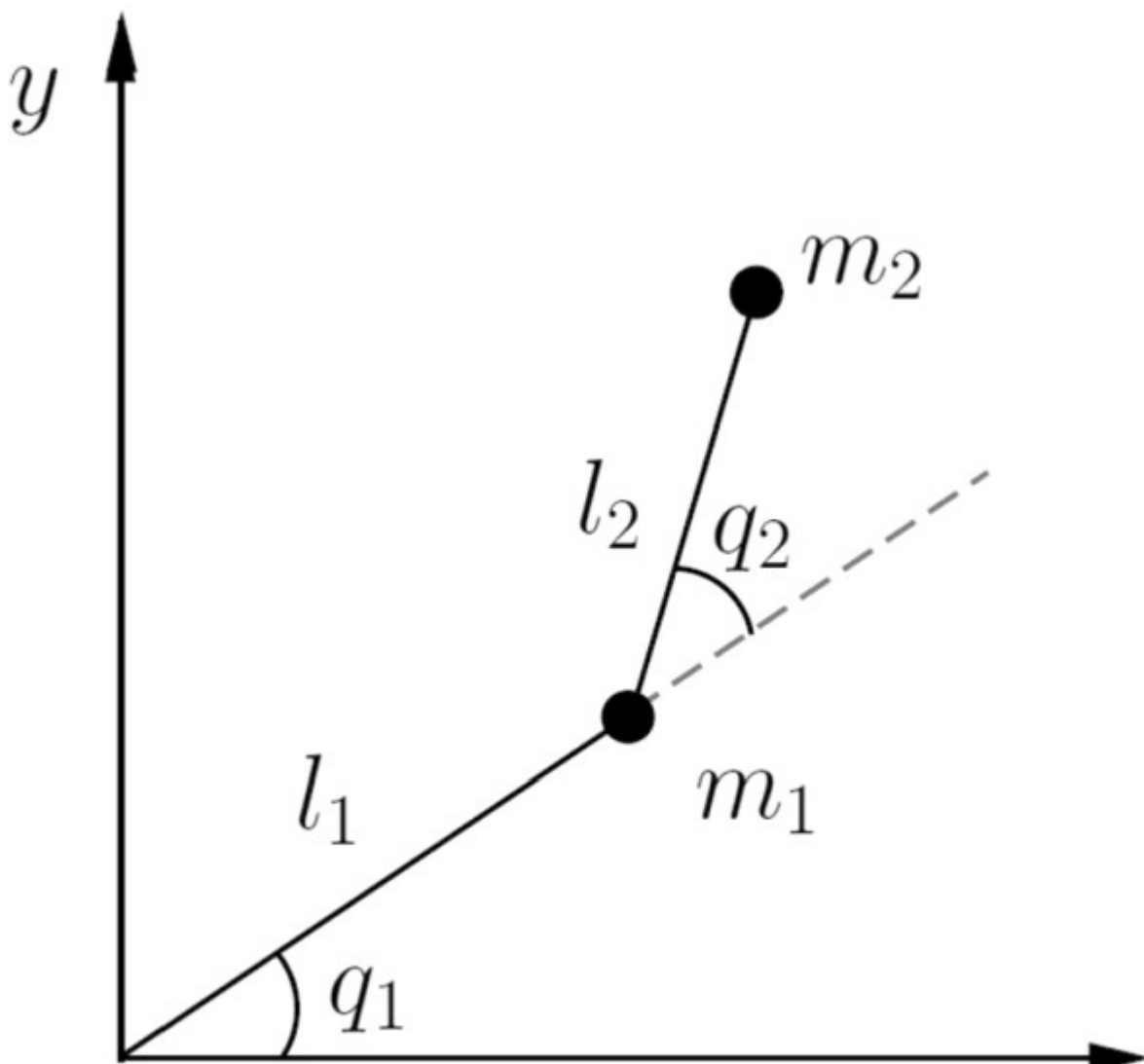
(18)

Once the estimation errors (\tilde{z}_1) and (\tilde{z}_2) go to zero, the sliding variable becomes $(s = \hat{s})$. Thus, the sliding variable goes to zero in finite time and the states converges to desired values in finite time [14]. Next the performance of proposed scheme is verified by simulation results.

4 Simulation results

To verify the effectiveness of proposed scheme, it is applied to a two link robotic manipulator affected by parametric uncertainty. The performance of proposed scheme is compared with a non-linear extended state observer and a sliding mode observer for the same level of parametric uncertainty. The simulations were carried out in MATLAB/ Simulink environment.

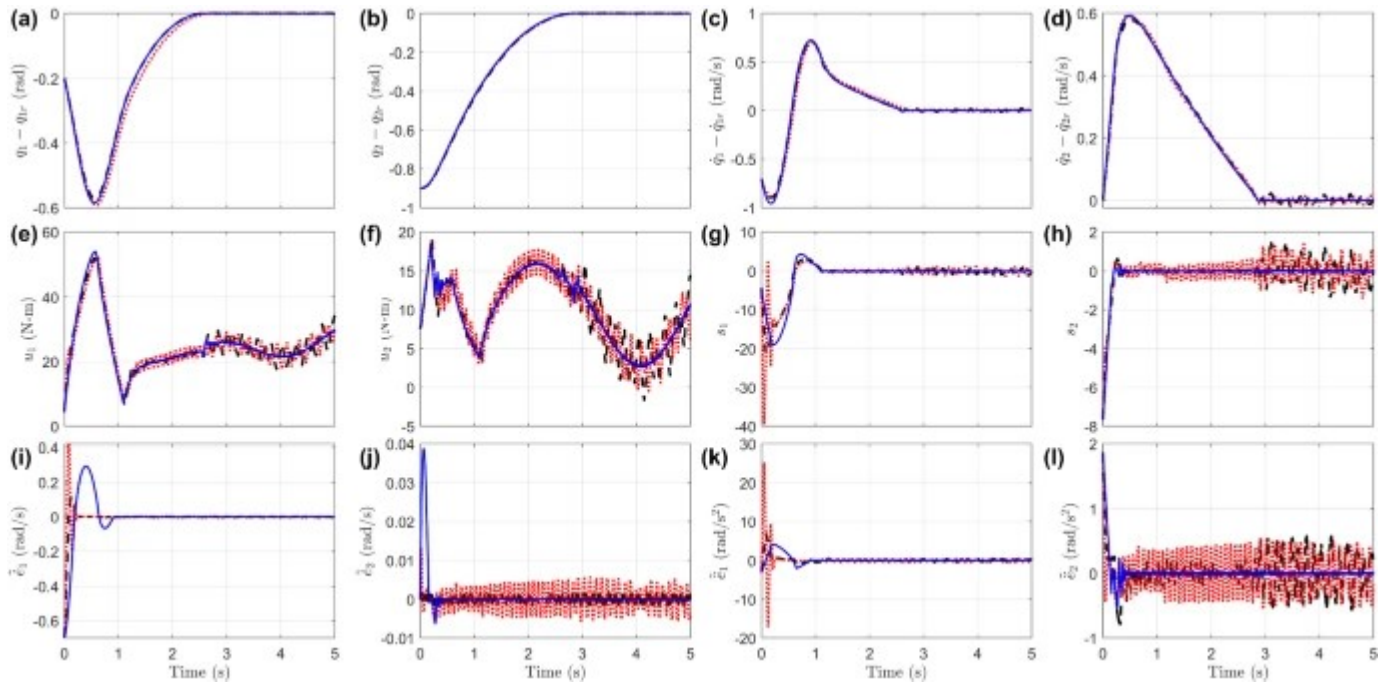
Fig. 1



$(0, 0)$ \bar{x}

Schematic of two link robotic manipulator

Fig. 2



Comparative plots of two links manipulator: NESO (red dotted line), SMO (black dashed line) and STO (blue solid line). (Color figure online)

The dynamics of two link manipulator shown in Fig. 1 is given by [21, 22]:

$$\begin{aligned}
 & \begin{bmatrix} M_{11}(q_2) & M_{12}(q_2) \\ M_{12}(q_2) & M_{22}(q_2) \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \\
 & \begin{bmatrix} B_{12}(q_2) \dot{q}_1^2 - 2B_{12}(q_2) \dot{q}_1 \dot{q}_2 \\ B_{12}(q_2) \dot{q}_2^2 \end{bmatrix} \nonumber \\
 & \quad + \begin{bmatrix} Q_1(q_1, q_2)g \\ Q_2(q_1, q_2)g \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}
 \end{aligned}$$

$$\end{aligned}}\$\$$$

(19)

where, $(\tau_1 = u_1)$ and $(\tau_2 = u_2)$ are the control inputs and the parameters are given by,

$$\begin{aligned} M_{11}(q_2) &= (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_1 l_1 l_2 \cos(q_2) + J_1 \\ M_{12}(q_2) &= m_2 l_2^2 + m_2 r_1 r_2 \cos(q_2) \\ M_{22} &= m_2 l_2^2 + J_2 \\ B_{12}(q_2) &= m_2 l_1 l_2 \sin(q_2) \\ Q_1(q_1, q_2) &= (m_1 + m_2)l_1 \cos(q_2) + m_2 l_2 \cos(q_1 + q_2) \\ Q_2(q_1, q_2) &= m_2 l_2 \cos(q_1 + q_2) \end{aligned}\$\$$$

The actual parameters for simulations were considered as: $(m_1 = 0.5)$ kg, $(m_2 = 1.5)$ kg, $(J_1 = 5)$ kg \cdot m, $(J_2 = 5)$ kg \cdot m, $(l_1 = 1)$ m, $(l_2 = 0.8)$ m and $(g = 9.81)$ m/s². The performance of the proposed scheme is compared with two non-linear observers i.e. a Non-linear Extended State Observer and a Sliding Mode Observer. The initial conditions of plant is set as $(q_1 = 0)$ rad, $q_2 = 0.2$ rad, $\dot{q}_1 = \dot{q}_2 = 0$ rad/s and reference trajectories are selected as: $(q_{1r} = 0.2 + 0.7 \sin(t))$ rad and $(q_{2r} = 0.3 + 0.6 \cos(1.5t))$ rad for all simulation results. The nominal parameters

The non-linear extended state observer (NESO) is implemented [23, 24] to estimate the derivatives of the states (\dot{q}_1) and (\dot{q}_2) as follows:

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 + \beta_1, g_1(\tilde{z}_1) \\ \dot{\hat{z}}_2 &= \beta_2, g_2(\tilde{z}_1) \end{aligned}\$\$$$

(20)

where, (\hat{z}_1) and (\hat{z}_2) are the estimates of $(z_1 = \dot{q}_i)$ and $(z_2 = \ddot{q}_i)$, for $(i = 1, \text{ and } 2)$. The non-linear function $(g_i(\tilde{z}_1))$ is defined as:

$$g_i(\tilde{z}_1) = \left\{ \begin{array}{l} \|\tilde{z}_1\|^{\gamma_i} \end{array} \right.$$

$$\begin{aligned} & \text{sgn}(\tilde{z}_1), \text{ and } |\tilde{z}_1| > \delta_i \frac{\tilde{z}_1}{\delta_i^{1-\gamma_i}}, \text{ otherwise} \end{aligned} \quad , i = 1, 2.$$

(21)

where $(\tilde{z}_1 = z_1 - \hat{z}_1)$, the parameters $(\delta_i > 0)$ and $(0 < \gamma_i < 1)$. The NESO parameters for the first and second links are selected as: $(\beta_1 = 10, \beta_2 = 100, \gamma_1 = 0.1, \gamma_2 = 0.1, \delta_1 = 0.05, \delta_2 = 0.1)$.

The Sliding Mode Observer (SMO) is implemented [25] as follows:

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 + l_{1i} \tilde{z}_1 + k_{s1} \text{sgn}(\tilde{z}_1) \\ \dot{\hat{z}}_2 &= l_{2i} \tilde{z}_2 + k_{s2} \text{sgn}(\tilde{z}_2) \end{aligned}$$

(22)

where, (l_{1i}, l_{2i}) are the linear gains and (k_{s1}, k_{s2}) are the switching gains of the SMO. The signals (\hat{z}_1) and (\hat{z}_2) are the estimates of $(z_1 = \dot{q}_i)$ and $(z_2 = \ddot{q}_i)$, for $(i = 1, \text{ and}, 2)$. The observer parameters are selected for the first and second links as: $(l_{11} = l_{12} = 10)$, $(l_{21} = l_{22} = 700)$, $(k_{s1} = 2.5)$ and $(k_{s2} = 60)$.

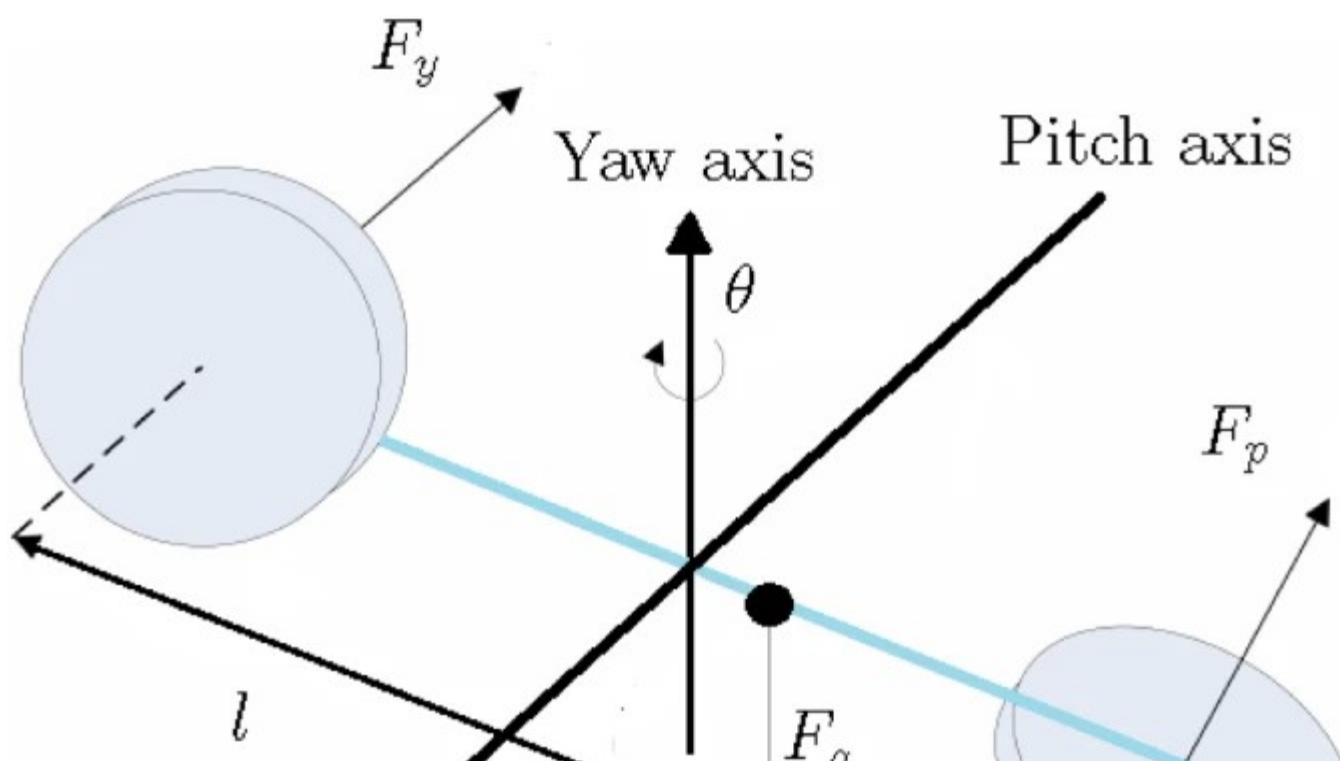
The super twisting observer is implemented as per (17) with $(k_1 = 10)$ and $(k_2 = 20)$ for both of the links. The sliding variables are selected as per (16) of the revised manuscript with $(c_1 = 50)$ and $(c_2 = 50)$ for the first link and $(c_1 = 20)$ and $(c_2 = 20)$ for the second link. The control inputs are selected as per (17) and (18) with $(\tau = 0.1)$, $(k_{dis} = 15)$ for both of the links. The non-linear functions are selected as: $(f_1(q, t) = \frac{B_{12}(q_2)}{\dot{q}_1^2 - 2B_{12}(q_2)\dot{q}_1\dot{q}_2\{M(q_2)\}})$ and $(f_2(q, t) = \frac{B_{12}(q_2)\dot{q}_2^2\{M(q_2)\}}{M(q_2)})$. The disturbances are considered as: $(\bar{d}_1(\vec{q}, t) = \frac{Q_1(q_1, q_2)g\{M(q_2)\}}{M(q_2)})$ and $(\bar{d}_2(\vec{q}, t) = \frac{Q_2(q_1, q_2)g\{M(q_2)\}}{M(q_2)})$, where $(M(q_2) = M_{11}(q_2)M_{22} - \{M_{12}\}^2(q_2))$.

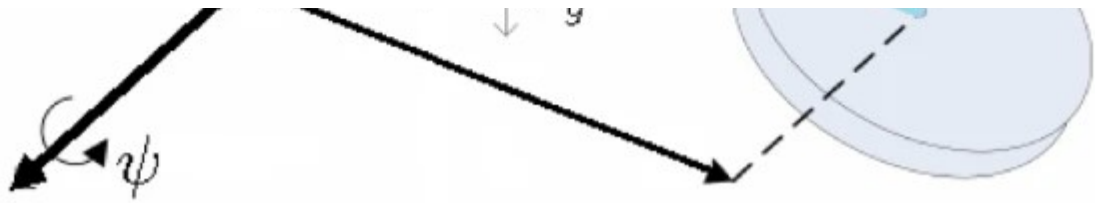
The comparative plots of the proposed scheme and the NESO and the SMO based control are shown in Fig. 2. The tracking errors of both links of the manipulator are shown Fig. 2a–d. All three methods are bringing the system states from their initial points to the desired reference trajectories. The control inputs plots are shown in Fig. 2e, f; it can be observed that the proposed scheme is free from chattering as compare to the other two methods. Similarly, the behavior of chattering can be observed in the sliding variables, as shown in Fig. 2g, h. The comparative plots of estimation errors are shown in Fig. 2i–l, in the steady-state chattering phenomenon, can be observed with a NESO and an SMO. The effect of chattered estimated states and their derivatives can be seen in the control inputs in the comparative results. As the magnitude of chattering in the estimated states increases, it induces chattering in the control inputs and eventually in the system states.

5 Experimental validation

The 2-DOF helicopter model is a highly non-linear system with inputs coupling. A free body diagram of the 2-DOF helicopter system is shown in Fig. 3. It consists of two Brush-less DC motors to control motions over the pitch axis and yaw axis.

Fig. 3





A free body diagram of 2-DOF helicopter system

5.1 Dynamics

The non-linear dynamic model of 2-DOF Helicopter system [26] is given by

$$\begin{aligned} \ddot{\theta} = & -\frac{B_p \dot{\theta} + m l^2 \dot{\psi}^2 \sin \theta \cos \theta + m g \cos \theta}{J_p + m l^2} + \frac{k_{pp}}{J_p} + m l^2 u_p + \frac{k_{py}}{J_p} + m l^2 u_y \end{aligned}$$

(23)

$$\begin{aligned} \ddot{\psi} = & \frac{2m l^2 \dot{\psi} \dot{\theta} \sin \theta \cos \theta}{J_y + m l^2 \cos^2 \theta} - \frac{B_y \dot{\psi}}{J_y + m l^2 \cos^2 \theta} + \frac{k_{yp}}{J_y + m l^2 \cos^2 \theta} u_p + \frac{k_{yy}}{J_y + m l^2 \cos^2 \theta} u_y \end{aligned}$$

(24)

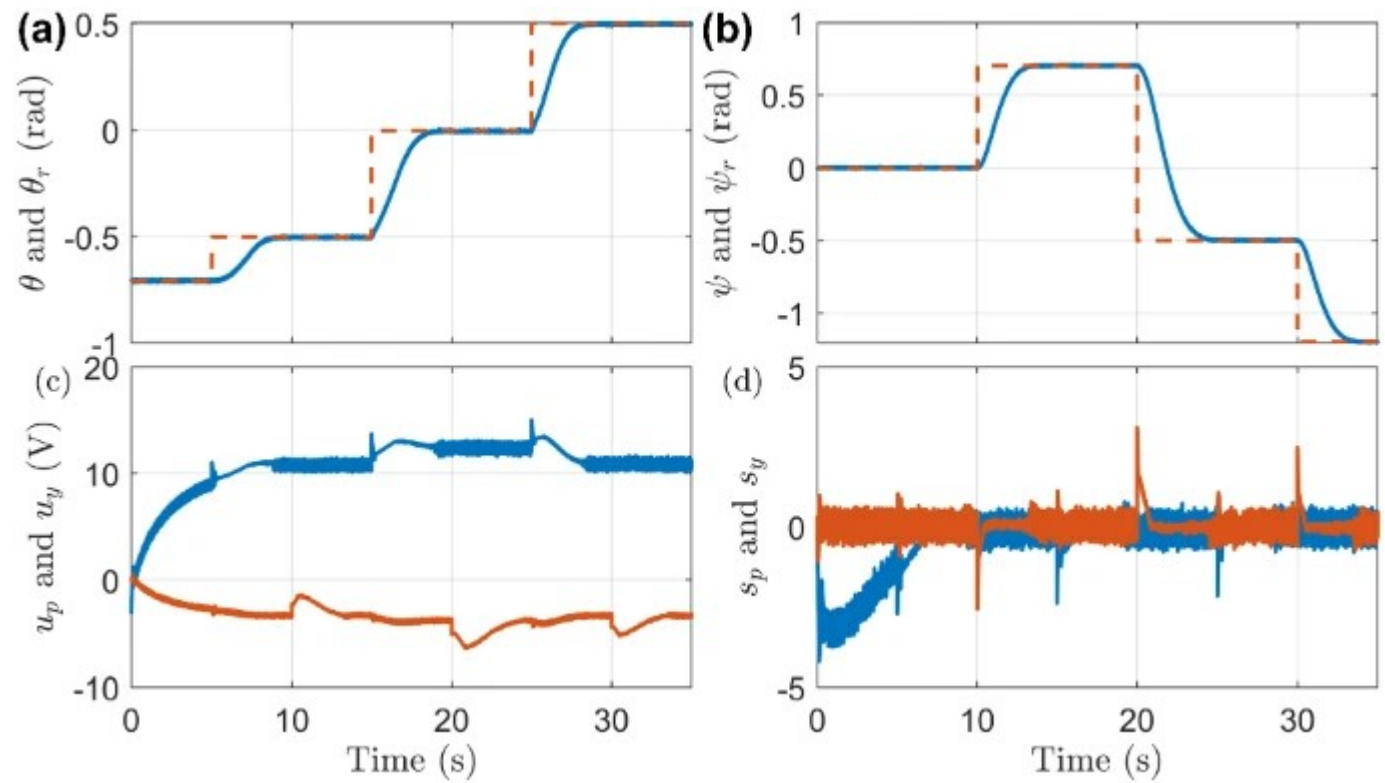
where θ and ψ are the pitch angle and yaw angle respectively. The motor voltages u_p and u_y are the control inputs to be designed. The control law is designed by considering the system as two single input single output systems. The objective to design control inputs u_p and u_y to track the desired pitch angle (θ_r) and the desired yaw angle (ψ_r) respectively.

Fig. 4



Laboratory setup of 2-DOF helicopter system

Fig. 5



Experimental results for the proposed scheme: a θ (solid line) and θ_r (dashed line), b ψ (solid line) and ψ_r (dashed line), c u_p (blue color) and u_y (orange color), and d s_p (blue color) and s_y (orange color). (Color figure online)

Table 1 Nominal parameters of 2-DOF helicopter system [26]

5.2 Hardware setup

The hardware setup is mounted on the fixed base for unlimited rotation around the yaw axis and 0.707 rad rotation along the pitch axis. It consists of two optical encoders to sense the pitch angle and the yaw angle. The optical encoders are connected to the data-acquisition card (Q2-DAQ). The commands given to the motors are amplified by the power amplifier (Volt-PAQ). The reference trajectories can be generated either by the joystick or the Simulink blocks. The Q2-DAQ and the Volt-PAQ are used to provide an interface between the hardware setup and a Personal Computer. The proposed algorithm is implemented in the MATLAB/Simulink, and then hardware-in-loop experiments were carried out at a laboratory, as shown in Fig. 4. The description of the system parameters and their nominal values are given in Table 1.

5.3 Controller design

The control inputs u_p and u_y are designed by considering the pitch and yaw dynamics as two sub-systems given in (23) and (24). The effect of input coupling is compensated in the control inputs by considering the input coupling acting as an external disturbance on the subsystems. The objective of the control law design is to bring the tracking errors $e_\theta = \theta - \theta_r$ and $e_\psi = \psi - \psi_r$ to zero in the finite-time. The control parameters are selected as: $c_1 = 3$, $c_2 = 4$, $\alpha_1 = 9/16$, $\alpha_2 = 9/23$, $\tau = 0.15$, $k_1 = 5$, $k_2 = 10$, $k_{\text{dis}} = 3$. The known non-linear functions are selected as:

$$\begin{aligned} f_p(x,t) &= \frac{B_p \dot{\theta} + m l^2 \dot{\psi}^2 \sin \theta \cos \theta}{J_p + m l^2} \end{aligned}$$

(25)

$$\begin{aligned} f_y(x,t) &= \frac{2m l^2 \dot{\psi} \dot{\theta} \sin \theta \cos \theta}{J_y + m l^2 \cos^2 \theta} \end{aligned}$$

(26)

for the pitch and yaw dynamics respectively. The other non-linear terms, the gravitation force and the difference between actual parameters and nominal parameters are considered as the disturbances acting on the individual channel.

5.4 Results

Figure 5 shows the experimental results obtained with the proposed controller. Figure 5a, b show the tracking performance of the pitch angle and the yaw angle respectively with the proposed scheme. Thus, the proposed scheme achieves the control objective to bring the tracking errors to zero. The control inputs (u_p) and (u_y) are designed in such a way that the change in a reference trajectory of one sub-system has minimal effect on the other sub-system, which can be observed from the plots of control inputs (u_p) , (u_y) and the sliding variables (s_p) , (s_y) are shown in Fig. 5c, d respectively. Thus, the proposed scheme successfully controls a non-linear uncertain system.

6 Conclusion

In this paper, the STO based FOSMC is implemented for an $(n-)$ th order non-linear uncertain system. Using STO, the estimation error goes to zero in finite time. It preserves the features of continuous control and finite-time convergence of the sliding variable and system states with a lower number of the sensor. The proposed scheme is compared with a non-linear extended state observer, and a sliding mode observer based FOSMC, and it shows better results in terms of smoothness of the control input and the tracking of the reference trajectories. The proposed scheme is validated on an experimental setup of the 2-DOF helicopter model.

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Ethics declarations

Conflict of interest

The authors declare that they have no conflict of interest.

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