Applied Mathematics and Computation
Volume 346, 1 April 2019, Pages 767-775

Properties of certain iterated dynamic integrodiffetential equation on time scales

Deepak B. Pachpatte

https://doi.org/10.1016/j.amc.2018.10.034

Abstract

The main objective of this paper is study the existence, uniqueness and other properties of solution of some iterated dynamic integrodifferential on time scales. The main tools employed are Banach Fixed Point theorem and an inequality with explicit estimates are used for proving our results.

Introduction

In last few decades notable contributions have been made in the theory of dynamical equations on time scales and its applications. The theory of time scale was initiated by [obscured] crete calculus [12]. Many [obscured] ications as approximation to partial differential equations. Dynamic Integral and Integrodifferential on time scales is found to be valuable phenomena in the field of Science and Engineering [10]. Integrodifferential equation with boundary conditions has applications in study of Population dynamics and optimal control problems [14].

The problems of Existence, Uniqueness and other properties of solutions for dynamic equations on time scales have been recently treated in using various techniques such as Leray Schauder, Brower theory and fixed point theorems [1], [2], [11], [15], [16], [20]. Basic of partial differential equations are given in [13].

Recently lot of attention has been devoted to the study of linear and nonlinear dynamic inequalities on time scales which are used in study of Dynamic equations and obtaining explicit estimates [17], [18], [19]. In [3], [4], [5], [9] authors have studied Wendroff type inequalities using Picard's operator and various type of other inequalities in two independent variables on time scales.

Motivated by the above results in this paper we investigate the existence, uniqueness and other properties of the iterated dynamic integrodifferential equation on time scales of the form$x^{\Delta}(t) = f\left(t, x(t), \int_{t_0}^{t} g\left(t, s, x(s), \int_{t_0}^{s} h(t,\tau,x(\tau))\Delta\tau\right)\Delta s\right), x(t_0) = x_0$, for $0 \le t < \infty$ where x is unknown function to be found $h: I_T^2 \times R^n \to R^n,\ g: I_T^2 \times R^n \times R^n \to R^n$, $f: I_T \times R^n \times R^n \to R^n,\ x_0 \in R^n,\ x^{\Delta}$ is the generalized delta derivative of x, t is form time scale T which is nonempty closed subset of R the set of real numbers and $I_T = I \cap T$.

Section snippets

Preliminaries

In this section we shall introduce some preliminaries and notations which are used throughout this paper. We assume that the time scale T has a topology that it inherits from the real number R with standard topology. We define the jump operators σ, ρ on T by the two mapping $\sigma, \rho: T \to R$ satisfying conditions $\sigma(t) = \inf\{s \in T: s > t\}, \quad \rho(t) = \sup\{s \in T: s < t\}$ .The jump operators classify the points of time scale T as left dense, left scattered, right dense and right scattered according to whether $\rho(t) = t$ or $\rho(t) < t$, σ…

Main results

Now first we prove the dynamic inequality which is used for study of various properties

Theorem 3.1

Let $x(t)$, $p(t) \in C_{rd}(I_T, R_+)$, $k(t,s) \in C_{rd}(I_T^2, R_+)$ and $q(t,s,\tau) \in C_{rd}(I_T^2 \times R_+, R_+)$ be rd-continuous function for $0 \le \tau \le s \le t < \infty$. If $x(t) \le c + \int_{t_0}^{t} \left[p(s)x(s) + \int_{t_0}^{s} \left\{ k(s,\tau)x(\tau) + \int_{t_0}^{\tau} q(s,\tau,\eta)x(\eta)\Delta\eta \right\} \Delta\tau \right] \Delta s$, for $t \in T$ where $c \ge 0$ is a constant, $k(t,s).x(s) \in C_{rd}(I_T^2, R_+)$ and $q(t,s,\tau).x(\tau) \in C_{rd}(I_T^2 \times R_+, R_+)$ then $x(t) \le ce_H(t,t_0)$, for $t \in T$ where $H(t) = p(t) + \int_{t_0}^{t} \left[k(t,s) + \int_{t_0}^{s} q(t,s,\tau)\Delta\tau \right] \Delta s$….

Proof

Define a function $z(t)$…

…

Estimates on the solution

Now in this section we present some basic properties of solutions of Eqs. (1.1) and (1.2) under suitable conditions

Theorem 4.1

Suppose that the functions f, g, h in Eq. (1.1) are rd-continuous and satisfy the conditions
$|f(t,u,v) - f(t,\bar{u},\bar{v})| \le p(t)|u-\bar{u}| + |v-\bar{v}|$,
$|g(t,\tau,v,w) - g(t,\tau,\bar{v},\bar{w})| \le k(t,\tau)|v-\bar{v}| + |w-\bar{w}|$,
$|h(t,s,v) - h(t,s,\bar{v})| \le q(t,s,\tau)|v-\bar{v}|$, *where p, q, k are as in Theorem 3.1. Let* $c_2 = \left| x_0 + \int_{t_0}^{t} f\left(s,0,\int_{t_0}^{s} g\left(s,\tau,0,\int_{t_0}^{\tau} h(t,\eta,0)\Delta\eta\right)\Delta\tau\right)\Delta s \right|$. *If $x(t)$, $t \in I_T$ is any solution of (1.1), then $|x(t)| \le c_2 e_H(t,t_0)$, where H is given…*

…

Continuous dependence

Now in this section we study the continuous dependence of solution of Eqs. (1.1) and (1.2) on the functions involved therein. Consider an equation (1.1)-(1.2) and the corresponding equation. $y^{\Delta}(t) = \bar{f}\left(t, y(t), \int_{t_0}^{s} \bar{g}\left(s,\tau,y(\tau),\int_{t_0}^{\tau} \bar{h}(\tau,\eta,y(\eta))\Delta\eta\right)\Delta\tau\right)$, $y(t_0) = y_0$, for $t \in I_T$ where $\bar{f}: I_T \times R^n \times R^n \to R^n$, $\bar{h}: I_T^2 \times R^n \to R^n$, $\bar{g}: I_T^2 \times R^n \times R^n \to R^n$ and y_0 is a given constant in R^n.

Theorem 5.1

Suppose that the functions f, g and h in Eqs. (1.1)and (1.2) satisfy the conditions (4.1)–(4.3). Let y(t) be a given solution of Eqs. (5.1)and (5.2) …

…

Application

Many times it is difficult to find explicit solution of equation of form (1.1). Integral inequality with explicit estimates is a powerful tool which can be used in studying dynamic equation on time scales. Now we give an example

Example 6.1

Consider linear dynamic equation$y^{\Delta}(t) = p(t)y(t),\ \ y(t_0) = y_0$,and the corresponding perturbed dynamic equation as

$x^{\Delta}(t) = p(t)x(t) + f\left(t, x(t), \int_{t_0}^{t} g\left(t, s, x(s)\int_{t_0}^{s} h(t, s, \tau, x(\tau))\Delta\tau\right)\Delta s\right), x(t_0) = y_0$,for $t, t_0 \in T$ where y_0 is given constant, $p \in \Re$, $f: I_T \times R^n \times R^n \to R^n$, $g: I_T^2 \times R^n \times R^n \to R^n$ and $h: I_T^2 \times R^n \to R^n$.

…

…

Acknowledgment

This research is supported by Science and Engineering Research Board (SERB, New Delhi, India), Project File No. SB/S4/MS:861/13.…

Recommended articles

References (20)

M. Adivar *et al.*

Qualitative analysis of nonlinear volterra integral equations on time scales using resolvent and lyapunov functionals

Appl. Math. Comput. (2016)

Existence of solutions to boundary value problems for dynamic systems on time scales
Math. Anal. Appl. (2005)

J. Gu *et al.*
Some new nonlinear volterraFredholm type dynamic integral inequalities on time scales
Appl. Math. Comput. (2014)

WangQ. *et al.*
Existence and stability of positive almost periodic solutions for a competitive system on time scales
Math. Comput. Simul. (2017)

S. Andras *et al.*
Wendroff type inequalities on time scales via picard operators
Math. Inequal. Appl. (2013)

D.R. Anderson
Dynamic double integral inequalities in two independent variables on time scales
J. Math. Inequal. (2008)

D.R. Anderson
Nonlinear dynamic integral inequalities in two independent variables on time scale pairs
Adv. Dyn. Syst. Appl. (2008)

M. Bohner, A. Peterson, Dynamic Equations on Time Scales, Birkhauser Boston/Berlin...

M. Bohner, A. Peterson, Advances in Dynamic Equations on Time Scales, Birkhauser Boston/Berlin...

C. Corduneanu
Integral Equations and Applications
Cambridge University Press (1991)

There are more references available in the full text version of this article.

Cited by (1)

Solutions for a class of Hamiltonian systems on time scales with non-local boundary conditions ↗

2022, Applied Mathematics and Mechanics (English Edition)

View full text